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Some Experiments in Dynamics, Chiefly on Vibrations

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IN Toronto we have been doing, for some years past, experiments in dynamics which call for a little more mathematics than those that are commonly called first-year experiments. They are in our second course of laboratory work on Mechanics and Properties of Matter and deal with vibrations, sometimes bringing in rather harder differential equations than a first-year man can tackle. The apparatus is so simple that the following brief descriptions, with references, will be adequate for any reader who wishes to duplicate the experiments.

1. Rocking of Bodies on Circular Bearings

A typical problem is stated by Lamb[1] as follows:—"A circular cylinder of radius a, whose mass center is at a distance b from the axis, rolls on a horizontal plane. This problem includes the case of a compound pendulum whose knife-edge is replaced by a cylindrical pin which rolls on horizontal supports."

He shows that, if the amplitude is small, the time of vibration T, of the forward and backward rocking movement, is given by

$$T=2\pi\left(\frac{k^2+(a-b)^2}{bg}\right)^{\frac{1}{2}},$$

where k is the radius of gyration of the whole body about an axis through the mass-center G parallel to the axis of the cylinder on which the body rolls, and g is the acceleration due to gravity (Figs. 1a and 1b). If L is the length of the equivalent simple pendulum, then we may write

$$L=[k^2+(a-b)^2]/b.$$

Lamb adds: "The results evidently apply to any case of a solid of revolution rolling parallel to a vertical plane of symmetry, at right angles to the axis."

Figure 1 (a) shows a portion of a cylinder, cut so that the mass-center G is displaced from the geometric center O of the cylinder. Figure 1 (b) shows a pendulum on a rounded knife edge. Assuming the well-known equation for the period of a compound pendulum, *viz.*, $T=2\pi[(k^2+h^2)/hg]^{\frac{1}{2}}$, where h is the distance of the mass-center from the point of support, Lamb's expression is fairly obvious if sufficient approximation is made for very small amplitude.

In the laboratory we have used the following bodies, shown in Fig. 2 and described in Table I. Some of these test-objects were picked up from the supply and junk shelves of the workshop and laboratory storerooms. The deduction of the expressions for L from Lamb's formula gives good practice to the student and tests his ability to deal with centers of gravity and moments of inertia.

In experimentation the test-objects were placed on a smooth table top, set rocking, and

[1] H. Lamb, *Dynamics*, 2nd edition (Cambridge University Press, London, 1923), Sec. 66, p. 188. The symbols used in the present article differ occasionally from those used in the sources quoted.

timed with a stop watch. Three sets of 10 vibrations are usually made. The values of a range from 3 cm up to 18 cm and the period is usually less than 1 sec. The agreement between theory and practice is fairly good (i.e., to within five percent). It is convenient to remember and use the locally correct equations, $T=2\pi(L/g)^{\frac{1}{2}}=0.2006L^{\frac{1}{2}}$ (in Toronto), so that the approximate expression $T=\frac{1}{5}L^{\frac{1}{2}}$ enables one to make a quick mental or slide-rule calculation.

We have also a number of incomplete circular bands cut from the same large thin-walled brass tube (diameter 30.6 cm, thickness 0.15 cm). They were originally made to show that when balanced on a knife edge and set in vibration as compound pendulums they have the same period irrespective of sectorial angle ($L=2a$). When inverted as in Fig. 3 and set rocking it is easy to show that $L=2a(\alpha/\sin\alpha-1)$, where α is the semisectorial angle.[2] Our method of finding α is to lay the complete band on a drawing board, mark its trace, find the center of the circle and then lay, in turn, the incomplete bands on the trace, mark their ends and measure α with a protractor.

With the radius given above the period of vibration varies from 0.5 sec with a band for which $\alpha=59°$ to 1.4 sec with a band for which $\alpha=120°$. We tabulate in separate columns: The band identification number, α (deg), α (rad), $\alpha/\sin\alpha$ (from tables), $2a(\alpha/\sin\alpha-1)$ or L, $L^{\frac{1}{2}}$, period (calculated), period (observed). Following the best laboratory practice, the student is encouraged to tabulate his observations whenever they lend themselves to such a mode of presentation. The complete experiment involves using the six brass arcs that form the set, first as compound pendulums and then as rockers.

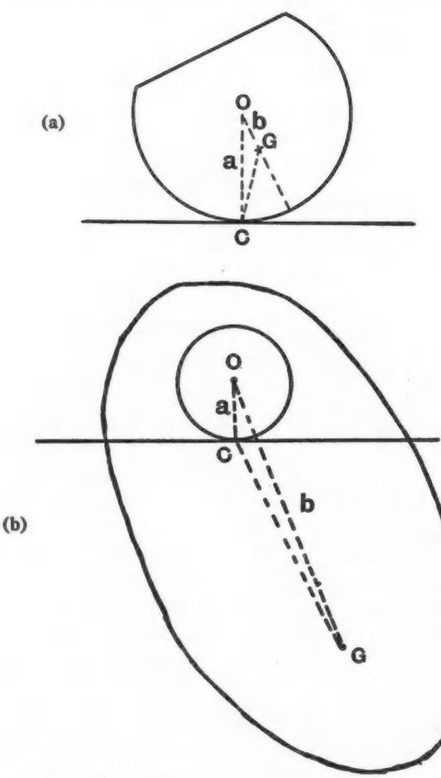

FIG. 1(a). A rocking incomplete circular cylinder. (b) A compound pendulum supported by a circular peg on a horizontal support and set in vibration.

2. Rocking of an Elliptic Plate

Lamb[3] sets the following exercise: "A cylinder, of any form of section, rocks on a horizontal plate making small oscillations about a position of equilibrium." Find the period. He shows that

$$L=(k^2+h^2)/(R-h),$$

where k is the radius of gyration of the cylinder about an axis through its mass-center perpendicular to the plane of rocking, h is the distance of the mass-center from the point of contact corresponding to the most stable position, and R is the radius of curvature of the curve at the same point of contact.

For an elliptic plate or disk or short cylinder—call it what you will—of semi-axes a and b (Fig. 4), $k^2=(a^2+b^2)/4$, $R=a^2/b$; hence[4]

$$L=\frac{b}{4}\left(\frac{a^2+5b^2}{a^2-b^2}\right).$$

We test this expression with a wooden elliptic plate which we found among the remains of an old set of geometric models. Its dimensions are $2a=37.7$ cm, $2b=24.6$ cm, thickness about 5 cm. The calculated period is 0.81 sec. The experimental period varied from 0.75 sec with small

[2] Note what happens at the two limits, $\alpha=0$ and $\alpha=\pi$.
[3] See reference 1, Sec. 66, p. 189.
[4] Note what happens at the limits $b=a$ and $b\ll a$.

amplitudes to 0.80 sec with large amplitude. The difference is possibly due to imperfections on the rocking surfaces.

3. Spiral Springs in Series and in Parallel

By spiral springs I mean what are more correctly designated as flat cylindrical springs. We have in the laboratory a large number of such springs that I have removed from old roller blinds in my modest housewifely spring-cleaning renewals. After these springs have been stretched so that their coils are nowhere in contact with one another they behave much better as far as Hooke's Law is concerned than most "boughten" or shop-wound springs. We make these roller-blind springs slightly different from one another by cutting off an inch or so of the length, finally bending over the ends of the wire to make loops for connections.

We hang these springs in turn from a support and make extension and vibration experiments and confirm the equation

$$T=2\pi(M/\lambda)^{\frac{1}{2}},$$

where λ = the force constant of the spring $=Mg/\gamma$, and γ = the extension of the spring produced by the load M (or the weight or force Mg). We find half-kilogram hooked masses ("weights" to the uninitiated) very convenient for this experiment. Since readings are to be comparative the force constant is calculated for a unit load of 1.5 kg and the periods are measured for the same load. This loading is suitable for the springs we use.

In the first part of the experiment we obtain the usual results obtained by first-year students, *viz.*, with increasing loads the extension is proportional to the load (Hooke's Law). From this we calculate a mean value of γ. Also the square of the period of up-and-down vibration is proportional to the load.[5] From this we get a mean value of λ. The student now obtains λ_1 and γ_1, λ_2 and γ_2, λ_3 and γ_3 for each of the three springs 1, 2, 3, supplied.

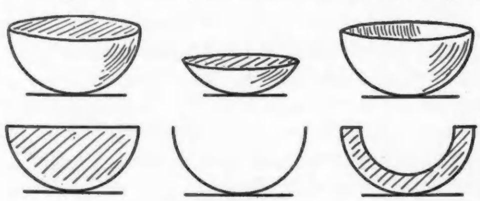

FIG. 2. Various laboratory objects that rock on circular arcs.

TABLE I. Bodies rocking on circular bearings of radius a.

Geometric shape	Source of supply	Value of L
Solid hemisphere	Half a billiard ball.	$1.733a$
Solid spherical cap	Cut from a billiard ball; height $a/2$.	$0.374a$
Thin hollow hemisphere	Copper bowl;[a] also glass evaporating dishes.	$1.333a$
Solid semicircle	Cut on the lathe, brass.	$(9/8\pi-2)a$, or $1.5343a$
Semicircular band	Half of a short length of thin-walled brass tube.	$(\pi-2)a$, or $1.1416a$
Thick semicircle	Cut on the lathe, radii a and b, brass.	$\frac{3\pi}{8}\frac{(3a^2+b^2)(a-b)}{a^3-b^3}-2a$

[a] Central Scientific Company, Cat. No. 75252.

In the second part of the experiment with the springs in series (Fig. 5a), we repeat the experiment, getting λ_s, γ_s, T_s, the subscript s denoting "in series." It follows from theory (including common sense) that

$$\gamma_s=\gamma_1+\gamma_2+\gamma_3, \tag{1}$$

and

$$1/\lambda_s=1/\lambda_1+1/\lambda_2+1/\lambda_3. \tag{2}$$

Therefore, with the same load (1.5 kg)

$$T_s^2=T_1^2+T_2^2+T_3^2. \tag{3}$$

The right-hand sides of Eqs. (1), (2) and (3) are calculated from the data previously acquired and compared with the "observation" values of γ_s, λ_s, T_s inserted in the left-hand sides. The agreement is fair.

In the third part of the experiment the springs are put in parallel (Fig. 5b) and the experiment is repeated to determine γ_p, λ_p, T_p, the subscript p denoting "in parallel." It follows from theory (and common sense) that

$$\lambda_p=\lambda_1+\lambda_2+\lambda_3, \tag{4}$$

$$1/\gamma_p=1/\gamma_1+1/\gamma_2+1/\gamma_3, \tag{5}$$

and with the same load (1.5 kg)

$$1/T_p^2=1/T_1^2+1/T_2^2+1/T_3^2. \tag{6}$$

Equations (4), (5), (6) are now verified by comparing the calculated right-hand sides with the observed left-hand sides. Again fair agreement is obtained.

[5] I omit the small correction due to the mass of the spring.

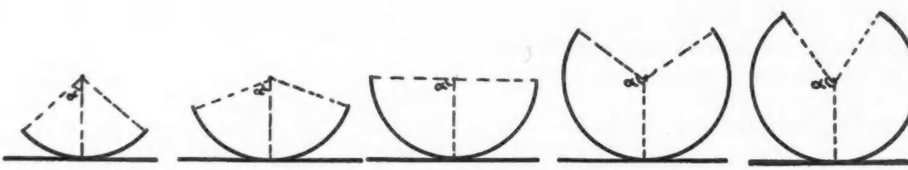

FIG. 3. Five incomplete circular bands standing on a table, ready for rocking. A sixth band is the complete circle not shown in the figure.

4. A Double-Ended Spring Experiment

Two nearly identical springs are attached by hooks to a hollow metal ball B (Fig. 6a) at the center, and at the far ends to the arms of a horizontal wooden framework about six feet long. The springs are under considerable tension so that when the ball is displaced to the right or left (Fig. 6b) both springs are always under tension. The longitudinal vibrations are therefore simple harmonic vibrations. This method of supporting a ball has an advantage over the ordinary vibration experiment—a load hanging from a spring—in that gravity does not have to be taken into account. The ball is hollow so that it does not cause the springs to sag appreciably.[6]

We shall assume the springs are equal in length and in force constants; if they are not, the equations below can be modified to represent the actual case. Let M be the mass of ball and λ the force constant of each spring. For longitudinal vibrations (Fig. 6b) the period T_l is given by $T_l=2\pi(M/2\lambda)^{\frac{1}{2}}$. Suppose now the ball is set in transverse vibration so that the springs move sideways, as in Fig. 6c, which is a plan of the apparatus. Let L be the unstretched length of each spring, L' the stretched length of each spring when the ball is at rest, and L'' the stretched length of each spring when the ball is at the extremity of a small transverse displacement of magnitude x. The central force is

$$2\lambda(L''-L)(x/L'')$$

$$=2\lambda x\left\{\frac{(L'^2+x^2)^{\frac{1}{2}}-L}{(L'^2+x^2)^{\frac{1}{2}}}\right\}$$

$$=2\lambda x\left(1-\frac{L}{L'^2+x)^{\frac{1}{2}}}\right)$$

$$=2\lambda x\left\{1-\frac{L}{L'}\left(1-\tfrac{1}{2}\frac{x^2}{L'^2}+\cdots\right)\right\}$$

$$=2\lambda x\left(1-\frac{L}{L'}+\frac{\frac{1}{2}Lx^2}{L'^3}\cdots\right).$$

For small values of x we may neglect the last term; hence

$$M\frac{d^2x}{dt^2}+2\lambda x\left(1-\frac{L}{L'}\right)=0,$$

or

$$T_t=2\pi\left(\frac{M}{2\lambda(1-L/L')}\right)^{\frac{1}{2}},$$

whence

$$T_l^2/T_t^2=L'/(L'-L).$$

The lengths can be measured and the periods observed. The confirmation is approximate only as the transverse vibrations are not really simple harmonic.

While the apparatus is in use it is interesting to pull the ball aside obliquely and watch its vibrations as it traces Lissajous' figures with varying phases. With a polished ball in a good light the appearance is very pretty.

5. The Double Pendulum

The double pendulum is an interesting arrangement of simple pendulums and is fairly well treated by Lamb.[7]

Using his notation and following him closely we may say the double pendulum consists of ball of mass m, suspended by a thin string of length l, supporting a ball of mass m' by a thin string of length l'. The vibrations are supposed to be confined to one vertical plane and to be of small amplitude. The vibrations may be of two types (Figs. 7a and b).

[6] If the ball or cylinder had a hole bored through it large enough for the spring to pass through it, the middle of the spring could be fastened to the ball and one spring only would be required.

[7] See reference 1, Sec. 44, p. 129.

If x and y are the horizontal displacements of m and m' from the vertical through O (Fig. 7), the tensions in the strings will be very nearly $(m+m')g$ and $m'g$ and the equations of motion are therefore

$$\left.\begin{aligned} md^2x/dt^2 &= -(m+m')g(x/l)+m'g(y-x)/l', \\ m'd^2y/dt^2 &= -m'g(y-x)/l'. \end{aligned}\right\} \quad (7)$$

Assuming that it is possible for m and m' to perform simple harmonic motions of the same frequency and coincident phases, we may write

$$x = A\cos(nt+\epsilon), \quad y = B\cos(nt+\epsilon), \quad (8)$$

whence by differentiation of Eq. (8), substitution from Eq. (7) and elimination of A and B we get

$$n^4-(1+\mu)(1/l+1/l')gn^2+(1+\mu)g^2/ll'=0, \quad (9)$$

where n is the frequency of vibration and $\mu = m'/m$. The roots of this quadratic are real and positive and one is greater than the greater of the two quantities g/l and g/l' and the other is less than the lesser of these two quantities. Denote the roots by n_1^2 and n_2^2. The periods are, therefore, $2\pi/n_1$ and $2\pi/n_2$.

In each mode of vibration the particles keep step with each other, but in one mode x and y have the same sign (Fig. 7a) and in the other mode x and y have different signs (Fig. 7b). Lamb calls these the normal modes of vibration. It will be noted that l and l' enter symmetrically in Eq. (9) so that the periods remain unaltered if the strings are interchanged. The value of μ depends on which is the upper (or lower) of the masses so that the periods are changed if the balls are interchanged. This is confirmed by experiment.

In our experiments, we use small brass balls each provided with two small hooks for suspension and attachment. The strings are lengths of fine fishing line also provided with hooks so that interchanging of balls and strings is quickly done. The pendulum is suspended from a fine steel hatpin driven into the end of a horizontal rod held in a tall clamp. The balls are weighed and the strings are measured. There is a little difficulty in the precise measurement of l and l'. The length of l is measured from the support to the center of the upper ball but one is undecided whether to measure l' downwards from the center of the upper ball or from its lower hook. The string should be practically non-stretching under the loads actually used. Denoting the longer and shorter strings by L and S respectively, and the larger and smaller balls by B and M respectively, the four arrangements showing the order of the strings and balls, LBSM, LMSB, SBLM, SMLB are tested by experiment and the observed results compared with the results calculated from Eq. (9).

In one of our experiments, $m = 107$ g, $m' = 29$ g, $\mu = 0.27$, $l = 71$ cm, $l' = 59$ cm. The arrangement was LBSM. Experiment gave periods of 2.0 sec and 1.1 sec for the two modes shown in Fig. 7a and Fig. 7b, respectively. Theory gives values of 10.8 and 27.2 for n^2 and, therefore, periods of 1.9 sec and 1.2 sec for the modes considered. When the arrangement was changed to SBLM the periods were unaltered.

The arrangements LMSB and SMLB were then set up. Here $\mu = 3.70$. Experiment gave periods of 2.28 sec and 0.52 sec. Theory gave the values 8 and 135 for n^2, whence the periods are 2.3 sec and 0.55 sec.

The experiment was repeated with balls of masses 71.0 g and 52.1 g, and strings of lengths 76.5 cm and 54.0 cm. The arrangements LBSM, SBLM gave observed periods of 2.10 and 0.97 sec and calculated periods of 2.06 and 0.94 sec. The arrangements LMSB and SMLB gave observed periods of 2.15 and 0.80 sec, and calculated periods of 2.15 and 0.78 sec.

Lamb proceeds to study two special cases depending on the limiting values of μ.

(*1*). *μ very small.* In this case Eq. (3) approximates to $n^4-(1/l+1/l')gn^2+g^2/ll'=0$ and therefore $n_1^2 = g/l$ and $n_2^2 = g/l'$, respectively. In the normal mode corresponding to the former of these (Fig. 8a), the upper ball oscillates almost like the bob of a simple pendulum of length l,

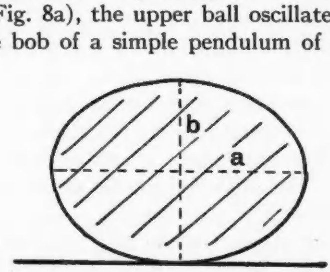

FIG. 4. An elliptical plate standing edgewise on the table.

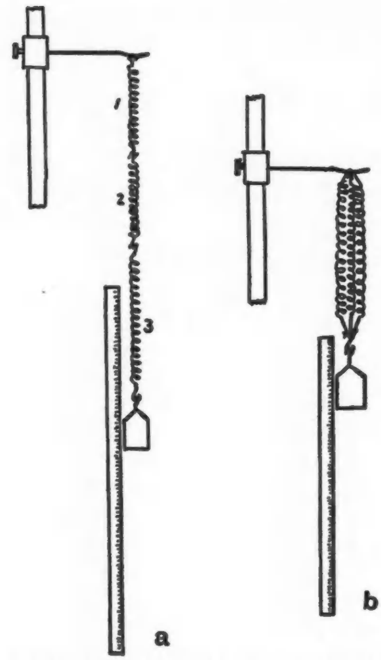

FIG. 5(a). Three spiral springs hanging in series and carrying a load; (b) the same, in parallel.

being only slightly affected by the smaller ball, while the latter behaves much like the bob of a simple pendulum of length l' whose point of suspension has a forced vibration of period $2\pi(l/g)^{\frac{1}{2}}$. In one experiment $m=71.0$ g, $m'=2.13$ g (and therefore $\mu=0.03$), $l=76$ cm, $l'=53$ cm. The observed period of both balls was 1.8 sec, and the calculated period, $0.20\ (76)^{\frac{1}{2}}$ or 1.74 sec.

With the strings interchanged the observed period was 1.43 sec and the calculated period $0.20\ (53)^{\frac{1}{2}}$ or 1.45 sec. In this normal mode the amplitude of vibration of the lower ball fluctuates considerably, a swing of almost zero value occurring about once in every four or five swings.

In the second normal mode (Fig. 8b) the upper ball remains comparatively at rest whilst the lower ball oscillates much like the bob of a simple pendulum of length l'. For example, with the same masses and strings as just described, when $l'=54$ cm the observed period of m' was 1.43 sec and when $l'=76$ cm the period was 1.79 sec. After many vibrations the amplitude of the big ball increases and this causes amplitude changes in the motion of the lower ball.

(2). *μ very large.* Equation (3) now becomes approximately $n^4-\mu(1/l+1/l')gn^2+\mu g^2/ll'=0$, and the roots can be shown to be $n_1^2=\mu g(1/l+1/l')$ and $n_2^2=g/(l+l')$, respectively. With the former root the lower ball is at rest while the upper ball vibrates like a particle attached to a string of length $(l+l')$ which is stretched between two fixed points with a tension $m'g$ (Fig. 9a).[8]

In one of our experiments, $m=2.13$ g, $m'=72.0$ g, $\mu=32.7$, $l=75$ cm, $l'=53$ cm. Experiment gave for the mode of Fig. 9a, a period of 0.22 sec. The calculated period is

$$2\pi\left(\frac{75\times 53}{32.7\times 980\times 128}\right)^{\frac{1}{2}},$$

which is 0.20 sec. In the second mode (Fig. 9b), the two balls are nearly always in a straight line, with the point of suspension, the lower ball oscillating like the bob of a pendulum of length $(l+l')$, the upper ball having very little effect. With $l=75$ cm and $l'=53$ cm, experiment gave a period of 2.28 sec while theory gives $0.20\ (128)^{\frac{1}{2}}$ or 2.26 sec.

The rest of Lamb's article on the double pendulum is interesting reading, especially the portion dealing with the fluctuating amplitudes of vibration. In the laboratory the students find good agreement between theory and practice and this experiment is one which they really enjoy, for it connects up their practical work with the differential equations of vibrations.

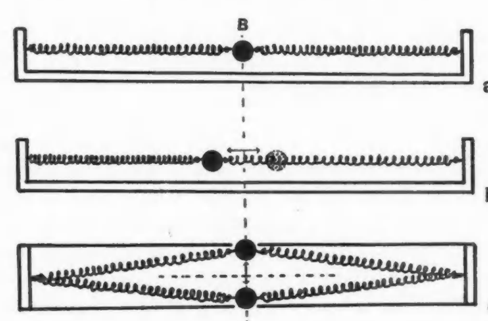

FIG. 6(a). A ball supported by two horizontal springs in tension; (b) the same, in longitudinal vibration, (c) the same, in transverse vibration.

[8] The proof of this is easy; it is, however, given in Lamb, see reference 1, Sec. 10, Ex. 3, p. 28.

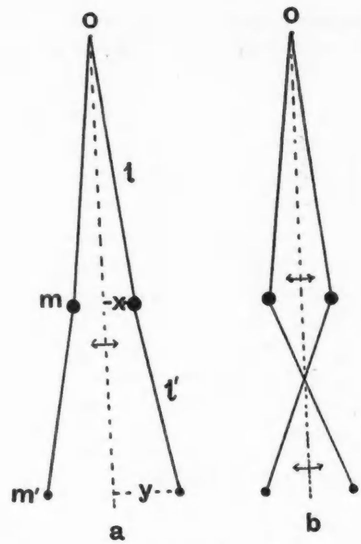

FIG. 7. The double pendulum showing, in general, the two normal modes of vibration.

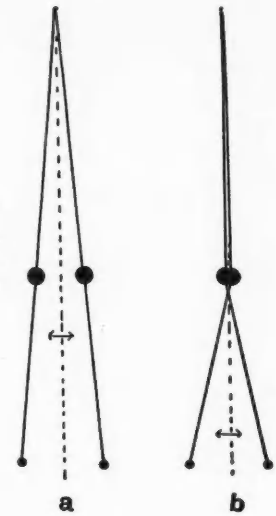

FIG. 8. The modes of vibration of the double pendulum when the mass of the upper ball is many times greater than the mass of the lower ball.

6. Transverse Vibrations of a Uniform Chain Hanging by One End

This problem is taken up by Lamb[9] who considers that a uniform flexible chain is suspended from one end and allowed to sway to and fro in a vertical plane. The fundamental mode of vibration and the first overtone are studied.

A preliminary attack by the theory of dimensions may be made. One would expect the period of transverse vibration to depend upon the length l of the chain, the mass of unit length ρ, and the acceleration of gravity g. Put $T=Cl^x\rho^y g^z$ where C is a numerical constant. Substituting the dimensions we get

$$[\mathrm{T}]=[\mathrm{L}]^x[\mathrm{M}/\mathrm{L}]^y[\mathrm{LT}^{-2}]^z,$$

whence

$$0=y,\quad 0=x-y+z,\quad 1=-2z.$$

Therefore $z=-\frac{1}{2}$, $x=\frac{1}{2}$ and $T=C(l/g)^{\frac{1}{2}}$, with a similar expression for the overtone.

This experiment is interesting for here we have the case of a nonrigid pendulum and the mathematical solution introduces the student to Bessel's functions.

Lamb assumes the vibrations are small so that vertical motion may be neglected. Assume an origin at the lower end of the equilibrium position (Fig. 10). Let ρ be the linear density of the chain. The pull at the point $x, y=g\rho x=P$, say. Consider an element of length δx. The horizontal components of the pulls on the lower and upper ends of the element δx are

$$-P\frac{dy}{dx}\quad\text{and}\quad P\frac{dy}{dx}+\frac{d}{dx}\left(P\frac{dy}{dx}\right)\delta x,$$

the derivatives being partial. Therefore the equation of motion is

$$(\rho\delta x)\left(\frac{d^2y}{dt^2}\right)=\frac{d}{dx}\left(P\frac{dy}{dx}\right)\delta x,$$

or

$$\frac{d^2y}{dt^2}=g\frac{d}{dx}\left(x\frac{dy}{dx}\right).$$

To ascertain the normal modes of vibration we assume, as usual, a solution of form $y=Y\epsilon^{i\sigma t}$, where Y is the maximum displacement as a function of x. Hence

$$\frac{d}{dx}\left(x\frac{dY}{dx}\right)+\frac{\sigma^2}{g}Y=0.$$

It is now convenient to replace x by a new variable. Put $x=\frac{1}{4}g\tau^2$, where τ is a time (see the next experiment where the meaning of τ is

[9] H. Lamb, *Higher Mechanics* (Cambridge University Press, London, 1929), Sec. 91, p. 225 and Sec. 95, p. 238.

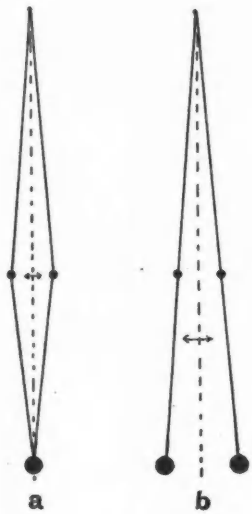

FIG. 9. The modes of vibration of the double pendulum when the mass of the lower ball is many times greater than the mass of the upper ball.

made clearer). On substitution and algebraical treatment we get

$$\frac{d^2y}{d\tau^2}+\frac{1}{\tau}\frac{dy}{d\tau}+\sigma^2y=0.$$

The solution, which is finite for $\tau=0$, is

$$y=CJ_0(\sigma\tau)\cos(\sigma t+\epsilon), \qquad (10)$$

where $J_0(\sigma\tau)$ is the Bessel function of zero order, *viz:*

$$J_0(\sigma\tau)=1-\frac{\sigma^2\tau^2}{2^2}+\frac{\sigma^4\tau^4}{2^2\cdot 4^2}+\cdots,$$

and C is arbitrary. The value of τ corresponding to the upper end $(x=l)$ is

$$\tau_1=(4l/g)^{\frac{1}{2}}=2(l/g)^{\frac{1}{2}},$$

and the condition that this end is fixed gives

$$J_0(\sigma\tau_1)=0. \qquad (11)$$

Equation (11) determines the admissible values of σ. The types of the corresponding normal vibrations are given by Eq. (10). The roots of Eq. (11) are (see Tables)

$$2.4048, \quad 5.5201, \quad 8.6537, \quad \cdots$$

the successive differences approaching the value of π, or as Lamb writes

$$\sigma\tau_1/\pi=0.7655, \quad 1.7571, \quad 2.7546, \quad \cdots$$

the successive terms tending to the form $(s-\frac{1}{4})$, where s is an integer. The longest period, therefore, is

$$\begin{aligned}T&=2\pi/(\text{the }\sigma\text{ corresponding to }\sigma\tau_1/\pi=0.7655)\\&=2\pi/(0.7655\pi/\tau_1)=2\pi\{2(l/g)^{\frac{1}{2}}\}/0.7655\pi\\&=(4/0.7655)(l/g)^{\frac{1}{2}}=5.225(l/g)^{\frac{1}{2}}.\end{aligned}$$

That is, $gT^2=27.3$ (ft), or $l/T^2=35.9$ (in Toronto), or $T=0.1669l^{\frac{1}{2}}$ (sec). This result may be compared with $T=0.201l^{\frac{1}{2}}$ for a simple pendulum and $T=0.164l^{\frac{1}{2}}$ for a uniform-rod pendulum suspended from its end.

In the modes after the first, the values of τ corresponding to the lower roots give the nodes or points of inflexion. Thus for the second mode there is a node at the point determined by

$$x/l=(\tau/\tau_1)^2=(0.7655/1.7571)^2=0.190,$$

and the period of this first overtone is given by

$$T'=(4/1.7571)(l/g)^{\frac{1}{2}}$$

or $gT'^2=5.18l$.

The two modes are represented on different scales in Fig. 11. The node N of Fig. 11b corresponds to the point of suspension of Fig. 11a.

In our laboratory experiment we use a common chain sold as No. 14 jack chain.[10] It is cut into

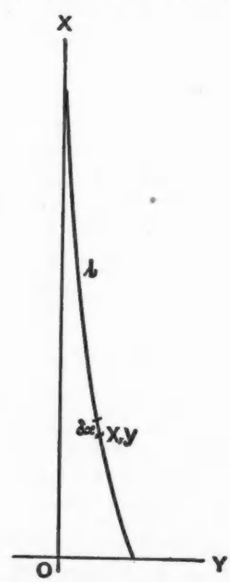

FIG. 10. The swaying suspended uniform chain.

[10] This is a steel chain of small smooth links, weighing about 12 feet to the pound.

TABLE II. Observations of a chain swaying in its fundamental mode.

Length l (cm)	Observed times of 10 vibrations (sec)	Period T (sec)	gT^2/l
180.0	22.3, 22.5	2.24	27.3
159.5	21.65, 21.05	2.105	27.2
140.9	19.85, 19.95	1.99	27.6
120.6	18.2, 18.4, 18.4	1.83	27.3
101.7	16.9, 16.8	1.69	27.4
79.8	14.9, 14.9	1.49	27.3
59.5	13.0, 12.9	1.295	27.6
40.8	10.6, 10.7, 10.7	1.067	27.3
		Mean	27.4

lengths of approximately 20, 40, 60 · · · 180 cm, and each length in turn is suspended from a projecting horizontal nail on the wall, set swaying in its fundamental mode and its period found. The tabulated values of gT^2/l agree well amongst themselves and with the theoretical value 27.3 Typical observations are recorded in Table II.

It is quite difficult to make the chain vibrate in its first overtone and many attempts may be necessary to get a good shape with a stationary node. A few results are given in Table III.

In their reports the students work out from theory and from mathematical tables the shape

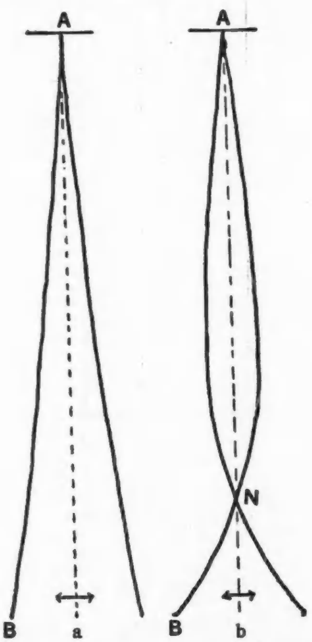

FIG. 11. The swaying chain (a) in its fundamental mode, (b) in its first overtone.

TABLE III. Observations of a chain swaying in its first overtone.

Length l (cm)	Distance of node from bottom (cm)	Calculated nodal distance $0.190l$ (cm)	Time of 10 vibrations (sec)	Period T' (sec)	gT'^2/l
180.0	34.0	34.2	9.9, 9.8	0.99	5.46
159.5	30.5	30.3	9.3, 9.2	0.925	5.26
140.9	26.5	26.7	9.0, 8.9 8.8	0.89	5.51
				Mean	5.4

TABLE IV. Predicted numerical information relating to a swaying chain.

x (cm)	x/l	$(x/l)^{\frac{1}{2}}$	$2.405(x/l)^{\frac{1}{2}} = \sigma\tau$	$J_0(\sigma\tau)$	$10J_0(\sigma\tau)$
0	0	0	0	1.000	10.00
20	1/9	0.333	0.802	0.845	8.45
40	2/9	0.471	1.133	0.704	7.04
60	3/9	0.577	1.388	0.574	5.74
80	4/9	0.667	1.603	0.453	4.53
100	5/9	0.745	1.793	0.341	3.41
120	6/9	0.816	1.965	0.242	2.42
140	7/9	0.881	2.121	0.155	1.55
160	8/9	0.943	2.269	0.073	0.73
180	1	1.000	2.405	0.000	0.00

of the chain. Thus for the fundamental mode with a chain of length 180 cm and assumed amplitude at the bottom of 10.0 cm, $\sigma\tau = 2.4048(x/l)^{\frac{1}{2}}$. The pertinent numerical quantities are as given in Table IV. From the numbers in the last column a careful plot is made to show the true shape of the chain in one of the extreme positions of its fundamental vibration (Fig. 11a). In the same way, for the chain vibrating in its first overtone, calculation of the values of $a[5.520(x/l)^{\frac{1}{2}}]$, (where a is the amplitude at $x=0$), for different values of x/l will enable the true shape of the chain to be realized graphically (Fig. 11b).

The students like this experiment with its practical introduction to Bessel's functions.

7. The Passage of a Transverse Wave Up and Down a Hanging Chain

This experiment is done immediately following the one just described. The usual formula for the velocity of propagation v of a transverse pulse along a horizontal wire stretched by a pull T is

$$v = (T/\rho)^{\frac{1}{2}},$$

where ρ is the linear density of the wire. If we may apply this expression to the hanging chain of Sec. 6 the velocity of a transverse wave up or

FIG. 12. A transverse pulse travelling up (and down) a hanging chain.

down the chain at the point x is given by $v_x = (g\rho x/\rho)^{\frac{1}{2}} = (gx)^{\frac{1}{2}}$ or $v_x{}^2 = gx$.

Now acceleration is expressed by vdv/dx which in this case is $g/2$; that is, a pulse travels up the chain with an acceleration $\frac{1}{2}\,g$ and travels down with a deceleration $\frac{1}{2}\,g$. Therefore the velocity on reaching the top of the chain is $(gl)^{\frac{1}{2}}$. The time taken[11] to travel from the bottom to the top is

$$\int dt = \int_0^l \frac{dx}{v} = 2\int \frac{dv}{g} = \frac{2}{g}(gl)^{\frac{1}{2}} = 2(l/g)^{\frac{1}{2}},$$

and therefore the time of the return journey is $4(l/g)^{\frac{1}{2}}$.

In the actual experiment a long chain is used. A sharp horizontal blow is given to the bottom of the chain (Fig. 12). The pulse travels up, is reflected at the top, comes down and gives the bottom of the chain a smart flip to the side, and travels up again. With luck as many as 10 return journeys can be observed.

In one experiment $l = 180$ cm and the time for 10 return journeys was observed to be 17 sec. The above theory gives, for the time of one return journey,

$$4(180/980)^{\frac{1}{2}} = 4(9/49)^{\frac{1}{2}} = 4(3/7) = 1.71 \text{ sec.}$$

For further reading on the oscillations of chains, reference may be made to *Dynamics of a System of Rigid Bodies*, by E. J. Routh.[12]

8. Up-and-Down Vibrations of a Hanging Chain Partly Counterbalanced by a Suspended Body

In Routh's *Rigid Dynamics* we find the following problem:[13] "A fine uniform chain is collected in a heap on a horizontal table, and to one end is attached a fine string which passes over a smooth pulley vertically above the chain and carries a weight equal to the weight of a length a of the chain. Prove that the length of the chain raised before the weight comes to rest is $a\surd(3)$, and find the length suspended when the weight next comes to rest."

Figure 13 shows successive measured positions of the chain. The experiment is interesting and

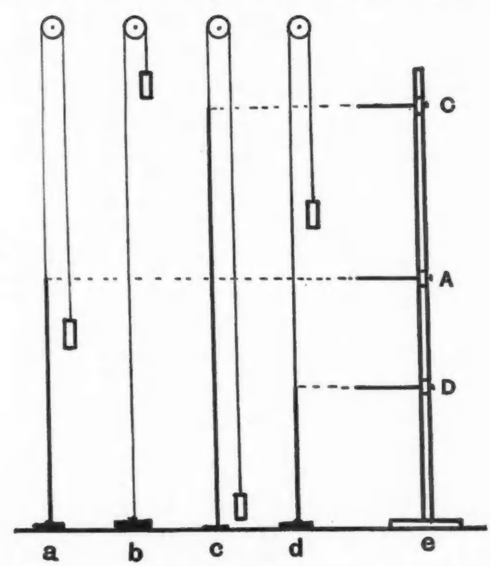

FIG. 13. A load supporting by a thread over a pulley a portion of a long chain; (a) The rest position of chain and load, (b) the start of the experiment, (c) the end of the first upward motion of the chain, (d) the position when the chain next comes to rest, (e) a tall stand fitted with clamps and rods by which the levels of the top of the chain at the stationary states are, after several trials, located.

[11] See τ_1 in the last experiment, Sec. 6.

[12] J. Routh, *Dynamics of a system of rigid bodies*, Ed. 5 (The Macmillan Company, 1892), Part II, Sec. 599 *et seq.*

[13] See reference 12, Part I, Sec. 300, p. 252, and Ex. 4, p. 253.

useful in that the theory involves simple differential equations requiring integrating factors and the student learns the practical use of this type of equation.

Theory: (1) To begin with, we consider only the first upward sweep of the chain. Let m = mass of unit length of chain, x = height of chain at some instant t in its first upward movement, $v = dx/dt$ = upward velocity of moving part of chain at time t, am = mass of the counterbalancing load. Using the momentum equation

(resultant upward force) × (element of time)
= change of momentum in this element of time,

the equation of motion is seen to be

$$g(ma-mx)dt = m(a+x)dv + (mdx)v.$$

After some manipulation we get

$$\frac{dv^2}{dx} + \frac{2}{a+x}v^2 = 2g\frac{a-x}{a+x}.$$

The integrating factor is $(a+x)^2$. Multiply by this factor and integrate. We find that

$$v^2(a+x)^2 = 2g\left(a^2x - \frac{x^3}{3}\right) + C.$$

When $x=0$, $v=0$; therefore, $C=0$,

$$v^2(a+x)^2 = 2gx(a^2 - x^2/3), \qquad (12)$$

and v will again be zero when $x = a\sqrt{3}$ (Fig. 13c).

It may be noted that the acceleration of the system, vdv/dx, is

$$\frac{g}{(a+x)^3}\left(a^3 - a^2x - ax^2 - \frac{x^3}{3}\right).$$

At the positions $x=0$ and $x=a\sqrt{3}$, the accelerations are g and $-g/3.72$, respectively.

(2) We consider now the first downward movement of the chain. The momentum equation is $gm(a-x)dt = m(a+x)dv - (mdx)v$, which leads to

$$\frac{dv^2}{dx} - \frac{2}{a+x}v^2 = 2g\frac{a-x}{a+x}.$$

The integrating factor is $1/(a+x)^2$, and on integration we get

$$\frac{v^2}{(a+x)^2} = 2g\left\{\frac{x}{(a+x)^2} - \frac{\sqrt{3}}{a(1+\sqrt{3})^2}\right\}. \qquad (13)$$

Therefore, the velocity will again be zero when $x = a/\sqrt{3}$ (Fig. 13d).

(3) For the second rise the constant of the integration leading to Eq. (12) is not zero. Substituting its new value we obtain a cubic equation for the height of the second ascent; and solving this by approximation we find the height to which the chain rises to be $1.369a$.

In our experiments (Fig. 13) we use a length of No. 14 jack chain, a light pulley placed about seven ft above the floor, cotton thread, and a narrow brass cylinder for the counterbalancing load. Care must be taken that cylinder and chain do not clash in their motions. A tall stand (Fig. 13e) carries three adjustable horizontal rods which are used to locate the "rest" positions of the top of the chain. Several determinations are made in order to get reliable averages. Sample results are given for a case in which $a = 88.6$ cm.

Chain rose to	148.8 cm,	whereas $a\sqrt{3} = 153.3$ cm;
Chain fell to	50.4 cm,	whereas $a/\sqrt{3} = 51.1$ cm;
Chain rose to	119.0 cm,	whereas $1.369a = 121.2$ cm.

The agreement is fair. The heap of chain on the floor is sometimes irregularly composed and this causes variations.

9. The Swaying Catenary

Just as a swaying chain does not remain straight, a catenary swaying at right angles to the plane of its rest-position does not remain coplanar, but a good approximation is obtained, if the amplitude of vibration is small, by assuming the catenary to be a rigid body. The experiment reminds the student to keep up his knowl-

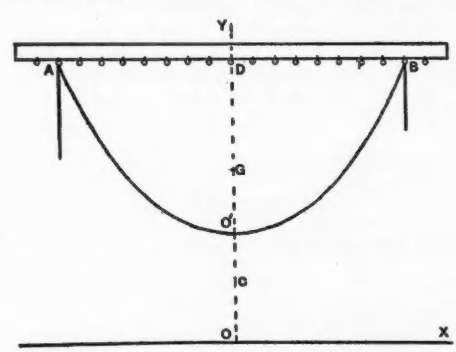

FIG. 14. The hanging uniform chain or catenary, showing the method of support.

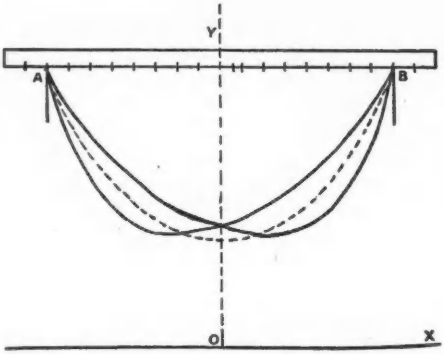

FIG. 15. The hanging chain or catenary vibrating to-and-fro in its own plane.

edge of the properties of this important curve. The equation of the catenary $AO'B$ (Fig. 14) with respect to its axes OX and OY is

$$y = c \cosh x/c$$
$$s = c \sinh x/c$$
$$y^2 = c^2 + s^2,$$

where c is the distance OO' and s is the length of the chain from the vertex O' to a point x, y on one side. If B is the point x_1y_1 and G is the position of the center of mass of the symmetrical chain $AO'B$, then

$$OG = \bar{y} = (cx_1 + sy_1)/2s,$$

where s is the length of the chain from O' to B, and the depth of G below AB is therefore given by

$$DG = (sy_1 - cx_1)/2s.$$

Crediting unit length of the chain with unit mass, the moment of inertia of the chain $AO'B$ about the axis OX is

$$2c^3\left(\sinh\frac{x_1}{c} + \tfrac{1}{3}\sinh^3\frac{x_1}{c}\right),$$

and about the axis AB,

$$2(c^2s + \tfrac{1}{3}s^3 - cx_1y_1).$$

Hence the period of vibration of the chain in the direction at right angles to the plane of its rest-position is given by

$$T = 2\pi\left(\frac{2(c^2s + \tfrac{1}{3}s^3 - cx_1y_1)}{(sy_1 - cx_1)g}\right)^{\frac{1}{2}}.$$

In the experiment it is easy to get x_1 and s but not easy to get c and y_1. If we draw a new axis of x through O' and denote the new ordinate of B by Y_1, we have

$$Y_1 = (y_1 - c) \quad \text{and} \quad c = (s^2 - Y_1^2)/2Y_1.$$

Since Y_1 is easily measured, c can be found, and therefore T, the period of vibration of a given catenary can be calculated.

In our experiments we again find it convenient to use No. 14 jack chain. A horizontal beam (Fig. 14) stretches above head-level across a corridor. It is provided with small hooks at every 5.0 cm so that changes in span, sag, and length of the catenary can be made quickly. In one experiment, span $AB = 2x_1 = 263.5$cm; length $2s = 458.0$ cm; $Y_1 = 169.7$ cm. The calculated value of c is 69.7 cm, whence T, calculated, is 2.28 sec. Observation gave 2.2 sec. For comparison we may note that the periods of a simple pendulum and a rod pendulum of length 169.7 cm are 2.61 sec and 2.13 sec respectively.

An additional exercise is to start the catenary swaying in its own plane from end to end as indicated in Fig. 15. The calculation of the period is too complicated for this paper. In fact, I do not think it has ever been done. Experiment with the catenary above gave a period of 1.80 sec.

At a later time I hope to describe other third-year experiments from our laboratory at the University of Toronto which may be of interest to American teachers. These include studies of a trifilar pendulum of easily adjustable length, the shape of a rotating liquid surface, the muzzle velocity of a bullet from an army rifle, a gravitation experiment giving the radius of the earth, a comparison of a vane anemometer with a Pitot tube, and spiral springs made of wires of different cross sections.

Physics Abstracting

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THE principal objective of the Study of Physics Abstracting[1] recently conducted by the American Institute of Physics was to obtain answers to the following questions:

1. For what purposes do United States physicists use abstracts of physics literature?

2. What is their opinion of the physics abstracting now available to them?

3. What, within reason, would they like to have in the way of physics abstracting?

The study was made primarily to provide the Joint Committee on Abstracting of the American Physical Society and the Institute with information upon which it might base sound and realistic recommendations for the improvement of abstracting in the field of physics. Related background material prepared during the project included an annotated List of Periodicals of Physics Interest[2] and an annotated and indexed List of Abstracting and Indexing Services of Physics Interest.[3] Since these lists have already been published, the present paper is limited to consideration of the survey of physicist opinion regarding the abstracting of the periodical literature of physics.

Mechanics of Survey

Physicist and technical librarian opinion on abstracting was obtained both by interview and by questionnaire circulation. Limitations of time, personnel, and funds, however, caused the interviews to be confined largely to individuals in the New York and Washington areas and made the questionnaire phase of the survey the principal source of information.

Keeping in mind the substantial prejudice that exists toward questionnaires in general, the staff and the committee decided that in this case (a) no information would be requested which could be obtained in any other way, (b) under no circumstances would the document exceed four pages in length, and (c) one of the four pages would be left blank to accommodate free-will, in-their-own-words comments by the respondents.

The physics questionnaire was sent to 2128 physicists selected at random from among those who had returned another questionnaire sent out some months previously in connection with a survey conducted jointly by the National Research Council and the editors of *American Men of Science*. This procedure made possible the use in the Abstracting Study of pertinent information collected in the earlier survey. Physicists returning the document numbered 1628, of whom 1477 filled it out essentially completely. Approximately 35 percent of those who returned questionnaires supplemented the check-off items with comments which varied from a plaintive "There are too many questionnaires these days" to extensive discussions and recommendations on numerous phases of abstracting.

All check-off answers were recorded on punched cards which already contained certain data taken from the earlier *American Men of Science* returns. The total information thus obtained made it possible to analyze the abstracting questionnaire results for the following breakdowns of physicist respondents:

1. All 1477 physicists taken together.
2. Each of four different age groups—25 and under, 26–35, 36–50, and over 50.
3. Each of nineteen subject specialist groups—biophysics, cryogenics, electricity and magnetism, electronics, fluid dynamics, geophysics, general physics, heat, infrared, light, mathematical physics, mechanics, molecular and atomic physics, nuclear

[1] The Study was supported under a contract with the ONR; active work extended approximately from September 1, 1948 to June 30, 1950. Negotiations for the contract were initiated under the general supervision of the Joint APS-AIP Committee on Physics Abstracting consisting of F. G. Brickwedde, H. H. Goldsmith (since deceased), George B. Pegram, Norman F. Ramsey, H. A. Robinson, G. A. Shortley, and Elmer Hutchisson, *Chairman*. After award of the contract, this Committee continued to serve in an advisory capacity.

[2] Published by Office of Technical Services, Department of Commerce, Washington 25, D.C., and is offered for sale by OTS under serial number PB-110,082.

[3] *Am. J. Physics* **18**, 274–299 (1950); this list also is available from OTS (see reference 2) under serial number PB-99951.

physics, physical optics, radiological physics, sound, spectroscopy, and underwater sound. (Division of physicists into these groups was based on the statement each had made in the *Men of Science* questionnaire regarding his own first competence.)

4. Each of three place-of-employment groups—educational institutions; government agencies; and industrial, business, and foundation laboratories.

To provide comparative data from the library field, approximately similar questionnaires were sent to some 300 technical and reference librarians of whom 202 returned answers.

Survey of Physicist Opinion

Results are presented below in the form of answers, based on questionnaire and interview data, to four specific questions, as follows:

1. What existing abstracting services do physicists use appreciably?
2. For what purposes do they use abstracts?
3. What do they think of the abstracting now available to them?
4. What would they like to have in the way of abstracting service?

Percentage figures in every case are based on the number of respondents in the category under discussion who answered that particular item. Although 1477 physicists filled out the questionnaire practically completely, different specific items were omitted, of course, by different numbers of physicists with the maximum variation being from about 1200 to well over 1400.

Use of existing abstracting services.—Physicists were asked to check all of a given list of abstracting services which they "use appreciably";[4] space also was provided for them to write in services not listed. The seven rating highest and the percentage of all respondents checking each are as follows:

1. *Physics Abstracts* (*Science Abstracts* A).................. 93 percent
2. *Chemical Abstracts*.......... 40
3. *Nuclear Science Abstracts*.... 28
4. *Electrical Engineering Abstracts* (*Science Abstracts* B).. 18
5. *Mathematical Reviews*....... 10
6. *Applied Mechanics Reviews*.. 7
7. *Engineering Index*.......... 7

Altogether, 33 services were checked or written in, although, in a number of cases, by but a very few respondents.

The age groups of 26–35 and 36–50 show no appreciable variation from all physicists taken together. For the group under 25, however, *Nuclear Science Abstracts* moved into second place while for those over 50 it dropped to fourth with *Electrical Engineering Abstracts* placing third. Variation of the separate subject categories from the over-all percentages was about what one would expect. For example, use of *Nuclear Science Abstracts* is highest among nuclear physicists and lowest among specialists in heat and thermodynamics; this kind of variation with field prevails throughout the subject groups. *Physics Abstracts* occupies first place (varying from 82 to 100 percent) in all categories except cryogenics where *Chemical Abstracts* received the most checks. *Chemical Abstracts* places second in "appreciable use" in eleven of the other subject categories and third or fourth in the remaining seven. *Nuclear Science Abstracts* is second in use in the areas of nuclear physics and radiological physics.

Among the three place-of-employment groups, order of appreciable use is the same as the over-all sequence for the four most-used journals, with some variations in order occurring in the next three.

Purposes for which abstracts are used.—Physicists were asked in the questionnaire to indicate the principal uses they make of abstracts according to two different bases of comparison. The first concerned employment of abstracts as a guide to original papers *versus* their use as a substitute for the complete articles. A few less than half (46 percent) checked "principally as a guide," six percent indicated their major use is "as a substitute for the original," and the remaining 48 percent checked "half as a guide and half as a substitute." On the use of abstracts as an aid in "keeping up" with the literature as compared with their employment "principally

[4] Quotation marks used here and in following pages indicate phrases quoted directly from the questionnaire items which the respondents answered.

for reference," 22 percent indicated the former as the major function of abstracts, 30 percent checked the latter and 48 percent said about half for each.

Opinion of present physics abstracting.—The journal *Physics Abstracts* (*Science Abstracts*, Section A), published in Great Britain by the Institution of Electrical Engineers, provides the only major abstracting service in English devoted wholly to physics. Also, essentially all members of the American Physical Society receive it by virtue of a Society arrangement with the publishers; that these scientists use it extensively is evidenced by the figures cited above. These facts made it clear at the beginning of the Study that any evaluation of present physics abstracting by physicists in this country necessarily would be to a considerable extent an expression of opinion about this particular publication. It seemed best, therefore, to recognize this fact frankly and relate one group of questionnaire items directly to *Physics Abstracts*. Dr. B. M. Crowther, editor of that journal, was consulted frequently throughout the Study and gave the staff particularly valuable assistance in framing this part of the questionnaire.

The principal query about the journal asked the physicists to rate it good, medium, or poor on a number of specific characteristics. The points considered and the results are shown in Fig. 1. It will be noted that the journal stands highest on the quality of its abstracts, with 96 percent of the respondents rating it either "good" or "medium"; this result correlates well with the consensus of physicists who were interviewed. The lowest composite rating is given on promptness of appearance of the abstract after publication of the original paper, and again correlation between questionnaire and interview opinion is close. The interviews also brought out the point, however, that excessive lag is a criticism leveled at essentially all abstracting services; the feeling of those interviewed seemed to be that on the average *Physics Abstracts* is little, if any, inferior to others in this respect. It is important to recognize in any discussion of promptness in abstracting, first, that any solution to the problem is limited by the existence of an inherent, irreducible minimum lag and, second, that the promptness with which an abstract can be published is closely related to the type of abstract and the manner in which it is prepared. These points are developed somewhat further at a later place in this paper.

Regarding breadth of coverage, questionnaire data appear somewhat more favorable to the journal than were the opinions expressed during personal interviews. In fact, both in interviews and in the freewill comments included with the questionnaires, recommendations were strong that the scope of coverage be broadened to include more applied physics; and that more journals, as well as a greater fraction of the contents of the journals, be abstracted. As is pointed out more fully later, however, a questionnaire item designed to delineate somewhat definitely the boundaries of the expanded coverage which was desired, gave little information of value. It should be noted in this connection that actually, the number of papers abstracted by *Physics Abstracts* has been growing steadily with a particularly large increase occurring from 1948 to 1949; figures for the last three years are as follows:

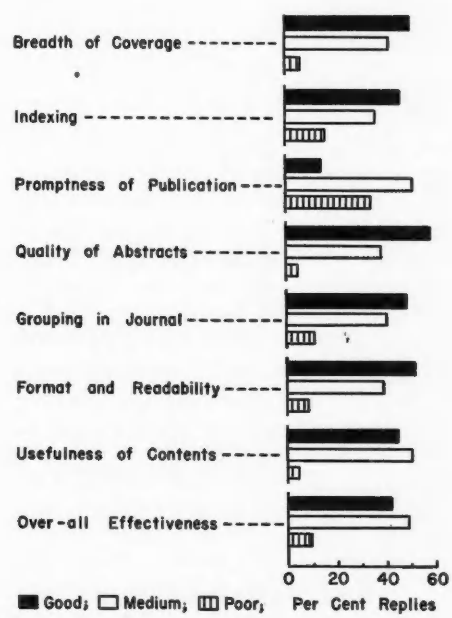

FIG. 1. Rating of various features of *Physics Abstracts* by physicists.

Year	Number of abstracts	Percent increase over preceding year
1947	3765	15.0
1948	4088	8.6
1949	7500	83.5

Physics Abstracts' indexing received the second largest number of "poor" votes while at the same time rating above average in the number of "good" checks. Interview and questionnaire comments indicate that physicists' unhappiness with this particular phase of the service stems principally from a desire for more cross references in the journal's annual indexes. On over-all effectiveness as an abstract journal, 42 percent of the respondents rated *Physics Abstracts* "good," 49 percent checked "medium," and nine percent graded it "poor."

Because the Universal Decimal Classification (UDC) numbers which accompany all abstracts in *Physics Abstracts* had been mentioned—invariably unfavorably—by a number of the physicists who were interviewed early in the Study, an item on this point was included in the questionnaire. Results were as follows:

1. Never use the UDC numbers 66 percent
2. Occasionally use them...... 28
3. Frequently use them........ 6

This information probably can be said to be more interesting than important in view of the fact that with the present format of *Physics Abstracts* there is no function of the journal which *requires* the reader to know anything about UDC numbers; their inclusion, however, is a decided convenience for those subscribers, particularly in Great Britain and on the continent, who are familiar with this classification system and like it.

For this portion of the questionnaire, results in the separate age, subject, and place-of-employment categories of physicists differed neither greatly nor significantly from those for the respondents as a whole.

Kind of Abstracting Desired

The various features of an abstracting service—such as breadth of coverage, promptness of publication, type of abstract, and authorship—are so interrelated that a decision with regard to any one may automatically place limitations on one or more others. Therefore, it seemed logical to begin the "kind of abstracting desired" portion of the questionnaire with an item on the relative importance of these factors.

Relative Importance of Various Features of Abstracts.—Results of the physicist ratings of five features of abstracts are shown in Fig. 2. Presence of "wide coverage" in first place is entirely consistent with the opinions expressed in interviews. Physicists frequently pointed out that a completely satisfactory abstracting service would permit the user to infer absence of publications in a given subject field from absence of abstracts in that field. Although they recognized the Utopian implications of this ideal, most "interviewees" urged as close as practical an approximation to it. No particular significance attaches to the figure of $5.00 in the last item. The cost factor obviously could not be completely ignored in an order-of-importance rating and this amount was chosen simply as a reasonable figure.

The "wide coverage" feature rated highest in importance in all of the twenty-six separate categories except biophysics, and in most cases by a wide margin. In general, the various subgroups also rated the features of "prompt publication," "abstracts by technical experts," and "extensive indexes" close together although with some variation in sequence. In all but five of the individual categories, the $5.00 cost feature placed last in importance.

There are certain interrelationships among these features which any realistic consideration of them must recognize. It would seem that "breadth of coverage" and "extensive indexing" should be fairly direct functions of cost. That is, other things being equal, the larger an abstracting journal's budget the more periodicals it could afford to abstract and the more indexers it could carry on the payroll. The factors of "promptness of publication" and "authorship of abstracts" are much less simply related to cost. With the former, there is an inherent, irreducible delay that is related much more directly to the methods of abstracting than it is to available funds, unless these are extremely large. In the case of author-

ship of abstracts, there may even be an inverse cost dependence if one assumes that an informative abstract written by a subject expert normally is superior to an indicative abstract written by a staff abstractor. The staff-written product of a nonexpert on the abstracting journal's payroll almost certainly will cost more per unit than abstracts obtained from a practicing scientist who does abstracting in his field of specialization as a sideline.

Promptness of publication and authorship are interdependent in that emphasis on one usually is purchased at the expense of emphasis on the other. In other words, the more an abstracting journal farms out the writing of abstracts to technical experts for whom this is not their principal activity, the greater will be the average delay in abstract publication. Staff-written abstracts are never out of the editor's control and ordinarily can be published relatively promptly.

Fields that should be covered.—Although agreement—both questionnaire and interview—was almost unanimous that wide coverage is of first importance and a large body of opinion urged strongly the broadening of *Physics Abstracts* coverage, no sharp, generally agreed-upon boundaries for this increased scope emerge from an analysis of questionnaire data. The respondent was given a list of sixteen borderline and associated fields and asked to check those he believed a physics abstracting service should cover; space was also provided for him to write in additional fields if he desired. Obviously it would have been a happy circumstance if almost everyone had checked a certain few fields and almost no one any of the others. That this did not happen is shown by the following tabulation of the percentage of respondents checking each of the listed areas:

	percent		percent
Chemical physics	71	Applied mechanics	29
Astrophysics	66	Astronomy	27
Biophysics	55	Meteorology	26
Teaching of physics	48	Mathematics (pure)	18
Medical physics	46	Electrical engineering	16
Geophysics	45	Applied aerodynamics	15
Applied electronics	44	Hydraulics	12
Computing methods	32	Mechanical engineering	9

In addition, 62 subject areas were written in on from one to eight questionnaires. The very wide difference of opinion among physicists as to just what constitutes adequate abstract coverage is indicated by the fact that among the fields written in were mineralogy, parapsychology, and wood technology.

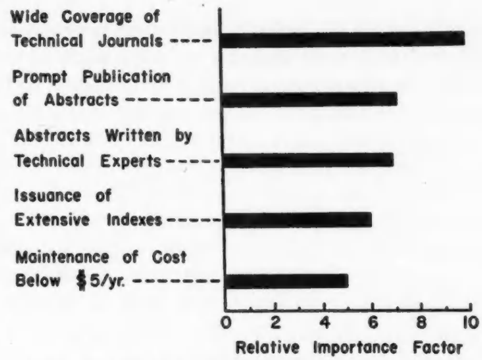

FIG. 2. Physicist's rating of relative importance of various features of abstracting.

Type of abstract desired.—In the questionnaire an indicative abstract was defined as one which tells the reader only whether he wants to read the original article and an informative abstract as one which summarizes the original paper's major arguments, data, and conclusions. Almost without exception the physicists who were interviewed stated that seldom if ever would they accept technical data and conclusions given in an abstract without themselves checking the original document and that, therefore, they actually required only abstracts which would tell them with certainty whether the paper contained material of interest to them. Consequently, it was rather expected that answers to an indicative-*versus*-informative questionnaire item would strongly favor the former.

This expectation was not fulfilled. Because cost of preparation of the two types might differ appreciably, opinion was sought both on an other-things-being-equal basis and on the assumption that informative abstracts would be the more costly. Eighty-three percent of the physicists who answered favor informative abstracts if the two are assumed to cost the same. More than half (58 percent) indicated a willingness to pay at least an additional $2.00 per year to obtain the informative type; and a third (34 percent) said they would pay as much as $5.00 more per year.

Actually, this lack of correlation between interview and questionnaire opinion probably is more apparent than real. There seems little doubt that almost all physicists use abstracts in an indicative fashion; that is, if they possibly can they consult the original paper before using its results. However, the original article occasionally is unobtainable. Also there is always the chance that the indicative abstract does not contain enough information for the reader to be absolutely certain whether or not he is interested in the paper. Finally, there is the chance that the indicative abstract may not contain sufficient information to permit adequate indexing. Therefore, it seems probable that the overwhelming questionnaire vote for informative abstracts reflects (a) a feeling of "the more information the better" even though the original will be read if available, (b) a desire to make quite certain that the abstract is a dependable guide, and (c) a concern that indexing be as complete as possible.

Miscellaneous.—A small but highly articulate segment of interview opinion supported the idea that extensive abstracting is unnecessary and that the needs of the physicist in this field would be met by a service which covered physics periodical literature exhaustively but gave only titles and references. To check how widespread this feeling might be an item on the point was inserted in the questionnaire. Results are as follows:

1. Favoring "reasonably informative abstracts with coverage limited to pure and semi-pure physics articles (i.e., about like abstracts and coverage of *Physics Abstracts*)" . . . . 86 percent
2. Favoring "titles and references only with very wide coverage of technical journals (i.e., essentially everything of physics interest with little or no attempt to discriminate between the important and the trivial)" 14

It has already been pointed out that the breadth of coverage attempted by an abstracting service probably is rather directly related to the cost of providing that service. One questionnaire item was devoted to an attempt to learn how much those desirous of very complete coverage would be willing to pay for it. Of more than 1200 physicists who answered this question, four-fifths indicated they would be willing to pay annually at least an additional $2.00, about two-fifths an extra $5.00, and less than a tenth (seven percent) $10.00.

No great enthusiasm was manifested for the scheme of printing abstracts on one side of the page only to permit easy maintenance of personal abstract files by "cutting and pasting." Three-fourths of the respondents who answered, either actively opposed the idea or said they personally would have no use for it; only about one-sixth indicated they want the service enough to pay anything extra for it.

Survey of Technical Librarian Opinion

The questionnaire sent to technical and reference librarians duplicated the physicists' document except for two or three items concerned specifically with library operations. Over-all agreement with the sample of physicist opinion discussed above was close but with a few interesting differences.

The seven abstracting services receiving the most "appreciable use" checks and the percentage of librarians checking each were:

1. *Chemical Abstracts* . . . . 97 percent
2. *Physics Abstracts* (*Science Abstracts* A) 91
3. *Engineering Index* 66
4. *Electrical Engineering Abstracts* (*Science Abstracts* B) . . 53
5. *Applied Mechanics Review* . . . 44
6. *Nuclear Science Abstracts* 44
7. *Technical Data Digest* 44

The variation here from the physicists' "first seven" probably is largely accounted for by the fact that most, if not all, of the technical librarians in the sample serve scientists in other fields in addition to physics. Some 25 other abstracting services were written in by from one to four librarians each.

Librarians assigned the following order of frequency to six different uses they make of abstracts of physics literature:

First: Locating specific references.
Second: Answering reference questions on a physics subject.
Third: Compiling bibliographies.

Fourth: Locating specific authors.
Fifth: Verifying references.
Sixth: "Keeping up" with material that should be acquired.

Librarians who rated the several features of *Physics Abstracts* agreed with physicists in assigning first and last places respectively to "quality of abstracts" and "promptness of publication." Librarians' ratings on "breadth of journal coverage," "indexing," and "over-all effectiveness" ran in the same sequence as in the physicists' case but from two to six percent lower in percentage of "good" checks. On the basis of these questionnaire results, technical and reference librarians in the United States seem to make little more use of Universal Decimal Classification numbers than do physicists; of librarians who checked this point, 62 percent checked "never use UDC," 26 percent said "occasionally," and only 12 percent indicated frequent use.

Regarding the desirable but interdependent features of an abstracting service discussed above, technical librarians rated "breadth of coverage" and "prompt publication" first and second in importance respectively just as did the physicists. Third and fourth places were interchanged, however, the librarians placing greater emphasis on "issuance of extensive indexes" than on "abstracting by subject experts." The cost-to-subscriber feature obviously was not applicable in this case.

Finally, the technical librarians were asked to check all of a given group of bibliographic features which they thought an adequate physics abstracting service should incorporate; results were as follows:

1. Inclusion of an author list with each issue	87 percent
2. Inclusion of a list of periodicals covered with the last issue of each volume	83
3. Inclusion of Union List symbols with rare titles to facilitate inter-library loan	59
4. Numbering of each entry for indexing use	52
5. Coverage of trade literature, Ph.D. theses, and other ephemera	47

Conclusions

When the Abstracting Study was initiated it was recognized that the general course of action which the Joint APS-AIP Committee subsequently would see fit to recommend might fall almost anywhere along a gamut running from "Make no changes whatever in the present system" to "Start a new physics abstracting service." The questionnaire and interview results described above seem to the Committee to warrant the basic conclusion that while physicists in the United States are not completely satisfied with the abstracting service they now receive, they are far from being sufficiently unhappy with it to warrant the establishment of a new abstracting journal, even if funds and personnel for such a venture were immediately available. Rather, it appears that any such action growing out of this Study should be directed realistically toward improving existing abstracting; use here of the word "realistically" can be thought of as implying that any recommendation which merely urges great expansion of one or more phases of the existing service but suggests nothing constructive as to how the expense of such extension is to be borne will be naive and unrealistic.

Although the details are not completely worked out as yet, expanded coverage and indexing are the points chiefly dealt with in the recommendations being discussed with the editor and publishers of *Physics Abstracts*. With regard to the former, it is hoped that a basis of operation can be achieved which will extend considerably the abstracting in a number of the borderline and associated subject areas with perhaps 100 percent coverage being guaranteed for a specific, limited list of journals. The objective in indexing is to increase appreciably the number of cross references and the average number of index entries per paper abstracted. The Council of the American Physical Society, the Committee and the Editor of *Physics Abstracts* are working together to develop means of support and mechanisms which will permit the attainment of these aims.

One other move which automatically should improve over-all physics abstracting brings the Institute publications into line with an international trend in abstracting. The composite picture presented by physicists' and technical librarians' opinion indicates that ideally one would like to obtain very promptly at practically no cost abstracts written by subject experts.

Certain elements of incompatibility in this combination already have been pointed out. The consensus, however, of both the London Royal Society Information Conference of June-July 1948, and of the Paris UNESCO Conference on Science Abstracting of June 1949, was that perhaps a reasonable and practical approximation to this ideal could be achieved through use of the often abused author abstract, if certain controls on it were to be set up. It seems clear that *if* every paper published in a technical journal were accompanied by an *adequate* author-written abstract, abstracts automatically would be available to abstracting journals with maximum promptness and at minimum cost. Also they would be written by subject experts; the commonly cited shortcomings of author abstracts are not based on charges of technical "inexpertness" but on alleged lack of objectivity, perspective, ability to judge significance, and abstracting skill. The conferences mentioned above suggested that the advantages of author abstracts be seized upon and the disadvantages minimized by setting up abstracting standards and educating authors to follow them. Specifically, it was felt that if editors of journals of primary publication could be persuaded (a) to require that all papers submitted for publication be accompanied by abstracts and (b) to take the same degree of editorial responsibility for the adequacy of the abstract that they now take for the quality of the paper, the ideal of combining low cost, prompt publication and technically expert authorship could be approached very much more closely than at present. In Great Britain considerable progress has been made in this direction during the past two years through an Abstracting Consultative Committee of the Royal Society which has drawn up specifications for the preparation of abstracts, or synopses, and has accomplished a great deal toward achieving the cooperation of journals of primary publication in following the kind of policy described above. The editors of the journals published by the American Institute of Physics have indicated their approval of this policy regarding abstracts and most of the journals are now following it; instructions for preparing abstracts in a style suitable for secondary use by abstracting services are included in the most recent Institute style manual.

Acknowledgments

The author wishes particularly to acknowledge the contribution made to the Abstracting Study by Mr. Robert S. Bray, the project librarian. Mr. Bray served on leave of absence from the Library of Congress and carried major responsibility in the compilation of the lists of periodicals and abstracting services of physics interest; he also rendered valuable aid in the preparation and analysis of the physicists' and librarians' questionnaires.

Contributions by nonproject people were too numerous to be described in detail in the space available here. It would not be proper to close, however, without at least mentioning a number of those whose freely given time and counsel were so helpful to the project staff. These include the entire Joint APS-AIP Committee of which Dr. Elmer Hutchisson, Dean of the Case Institute of Technology, is chairman; Dr. H. A. Barton, Director of the American Institute of Physics; Commander Oscar E. Hagberg and Dr. Julian F. Smith of the Scientific Information Division of the Office of Naval Research; Dr. R. C. Gibbs, Chairman of the Division of Mathematics and Physical Sciences, National Research Council; Mr. Verner W. Clapp, Chief Assistant Librarian, Library of Congress; Dr. Mortimer Taube, Assistant Chief, Technical Information Branch, Atomic Energy Commission; Mr. Theodore Vorburger, Comptroller, American Institute of Physics; Dr. Herbert A. Toops, Professor of Psychology, The Ohio State University; Dr. W. J. Eckert, Director of Pure Science, Watson Scientific Computing Laboratory; Dr. M. H. Trytten and Dr. C. J. Lapp, Office of Scientific Personnel, National Research Council; Dr. E. J. Crane, Editor of *Chemical Abstracts;* Dr. Ralph Hogan, Human Resources Division, Office of Naval Research; and Mr. George A. Freeman, Tabulation Services Branch, Executive Office of the Secretary of the Navy. The helpful assistance of Dr. B. M. Crowther, Editor of *Physics Abstracts*, has already been mentioned.

Michelson at Annapolis

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In a recent issue of the *American Journal of Physics*, Professor R. S. Shankland[1] of the Case Institute of Technology has given a very complete account of Albert A. Michelson's researches at Case from 1882 to 1889, together with a brief, but very accurate, summary of Michelson's work before and after his years at Case. Since Albert Michelson holds a unique position in the history of physics in America, it seems fitting to present a more detailed account of his years of naval service, in particular of his years at Annapolis, first as a student and later as an instructor. Professor Michelson's place in American physics is unique in that he was the first American to receive the Nobel prize. Other tangible recognitions of his genius, such as the Rumford Medal, the Copley Medal, the Franklin Medal, the Duddell Medal, etc., are too numerous to mention here. Professor Michelson's record is also unique in that he was the first professor of physics at two of the nation's most outstanding universities, the Case Institute of Technology and the University of Chicago. It is interesting to note that, while Michelson received eleven honorary degrees, he never completed the formalities of obtaining any "earned" degree, as the U. S. Naval Academy was not authorized to grant the bachelor of science degree until shortly after his death. It is indeed fortunate that Professor Michelson made his outstanding contributions to physics blissfully unaware of the fact, since discovered by the accrediting associations, that he was not qualified to hold any of the academic positions in which he served with such distinction.

Albert Michelson was admitted to the Naval Academy on June 28, 1869, age 16 years 6 months. The story of what Michelson in later years liked to call his "illegal" appointment to the Naval Academy has been told by Professor Shankland and others and will not be repeated here. Of the 86 midshipmen who entered the Academy in the summer of 1869, 29 graduated in June 1873, a mortality rate which shows the rigor of the course at that time. Then, as now, the relative standing of midshipmen in all subjects was published[2] for all interested persons to see. Midshipman Michelson ranked ninth in his class in over-all standing, and also ninth in total number of demerits, having been charged with 129. That Michelson's abilities and interests pointed him toward a career as a physicist, rather than as an officer of the line, might have been forseen even then for he ranked first in the course in optics and acoustics, but twenty-fifth in seamanship. Corroborating this are his standings of second in mathematics, third in chemistry, second in dynamics, third in statics, and second in heat and climatology. The brevity of Michelson's scientific papers has been commented upon by Millikan.[3] This quality appears to have been present in his undergraduate days

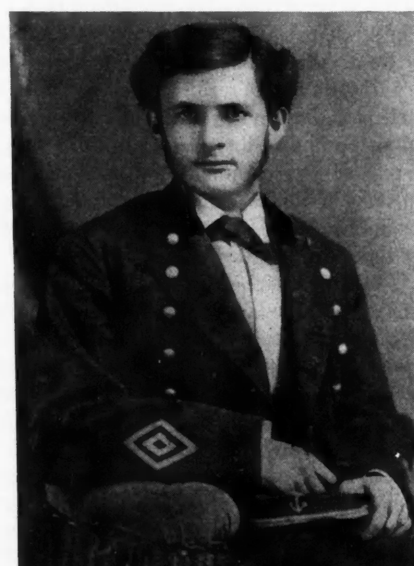

Fig. 1. Midshipman Albert A. Michelson, USN (1873).

[1] R. S. Shankland, *Am. J. Physics* **17**, 487 (1949).

[2] *United States Naval Academy Registers, 1869–73*, Government Printing Office, Washington, D. C.

[3] R. A. Millikan, "Biographical memoir of Albert Abraham Michelson," *National Academy of Science Biographical Memoirs*, Vol. 19, 4th memoir, Washington (1938).

for he ranked near the top of his class in technical grammar, but near the bottom in the course in history and composition, which the midshipmen of today have renamed "Bull."

The physics course at the Naval Academy between 1869 and 1873 consisted of five separate courses offered during the last two years. The textbook was Ganot's *Physics* (Atkinson's translation). The head of the Department of Natural and Experimental Philosophy was Lieutenant-Commander William T. Sampson, who many years later commanded the American Naval Forces at the Battle of Santiago, and for whom the building which now houses the physical sciences at the Naval Academy is named. It is interesting to note that the first two questions on the examination in optics for May 1873, in which Midshipman Michelson led his class, were: (1) "Discuss the undulatory theory of light. What fact proves the emission theory false?", and (2) "Describe Foucault's apparatus for determining the velocity of light."

Midshipman Michelson achieved his high class standing without being what is characterized in student slang as a "grind." On the contrary Rear Admiral Fiske,[4] who was a contemporary at Annapolis, recalls in his memoirs that Michelson had a reputation of studying less than any midshipman in his class. Admiral Fiske had good reason to remember Michelson. During a formation, in which Michelson was acting as officer-in-charge of his group, he made what Fiske thought was unjustified criticism of the latter's speed in responding to a command. Seeking to atone for this, Fiske challenged Michelson to a boxing match, which at that time was the accepted way to settle differences between classmates. Fiske was apparently unaware of the fact that Michelson was the school's lightweight champion, but his eight days on the sick list following his defeat caused him to remember this many years later. One gathers that Admiral Fiske found occasion to take pride in having been knocked out by so great a scientist, for he says that "His name will be remembered long after the names of many men who are eminent now have passed into oblivion."

Boxing was not the only extracurricular interest of Midshipman Michelson. He took special lessons in fencing and was one of the most skillful in his class in this ancient sport. In drawing he ranked first in his class, and it was during his midshipman days that he began painting, a hobby which he continued with increasing skill throughout his life. Music was still another of Michelson's diversified interests and he attained considerable skill on the violin.

On May 31, 1873, Midshipman Michelson was detached from the Naval Academy to await orders and on September 18, 1873 he was ordered to the *USS Monongahela*. The two years of sea duty which followed seems to have been a course in familiarization with ships for on October 31, 1873 he was detached from the *Monongahela* and on December 12, 1873 he was ordered to the *Minnesota*, only to be transferred on December 23, 1873 to the *Roanoke*. On August 19, 1874 he was transferred to the North Atlantic Squadron and joined the *Colorado*, but was almost immediately transferred to the *Worcester*. In July 1874 Michelson was promoted to Ensign and on December 15, 1875 he was ordered back to the Naval Academy as an instructor in physics. Except for a summer cruise on the *Constellation* in 1877, Ensign Michelson remained in this assignment until 1879.

While preparing for a lecture on optics in November 1877, a modification of Foucault's method for measurement of the velocity of light "suggested itself" to Ensign Michelson. Prior to this time three sets of direct measurements had been made: by Foucault using a rotating mirror as first suggested by Arago, and by Fizeau and Cornu using in each case a toothed wheel. The most serious objection to Foucault's method was the small magnitude of the deflection which led to a rather low degree of accuracy. The modification is clearly described by Michelson[5] in his paper presented at the St. Louis meeting of the American Association for the Advancement of Science in August, 1878: "It will be observed that the difference between this arrangement and that of Foucault is that the concave mirror is dispensed with, its office being accomplished by a lens and plane mirror; and that this arrangement permits the use of *any* distance between the mirrors."

During the winter of 1877–78, preliminary experiments were performed using "only apparatus such as could be adapted from the apparatus in

[4] B. A. Fiske, *From midshipman to rear admiral* (Century), p. 15.

[5] A. A. Michelson, *Proc. AAAS* **27**, 71 (1878).

the laboratory of the Naval Academy." In the spring a revolving mirror was built "at the expense of $10," and in May, 1878, using a distance of 500 feet, a deflection was measured which was twenty times that obtained by Foucault. Brief descriptions of these early experiments can be found in several sources.[6]

In July, 1878 the sum of $2000 was provided by "a private gentleman" for equipment which would permit carrying on the experiment on a larger scale. The "private gentleman" was Mr. A. G. Hemingway of New York, the father of the first Mrs. Michelson. (In 1877 Ensign Michelson married Miss Margaret McLean Hemingway, whom he met while she was visiting in Annapolis at the home of her uncle, the aforementioned Admiral Sampson.) After several months spent in the construction of apparatus and the making of "trial runs," the first of the final set of observations was made on June 5, 1879. All previous data were discarded, but after this date not a single observation was omitted. These experiments seem to have attracted considerable attention for, even before the start of the final observations, a leading New York newspaper[7] in a column titled "Science for the People" described the procedure to be followed and announced that: "The entire apparatus is understood to be finished and observations are shortly to begin at Annapolis." Thus at the age of 26 Ensign Michelson, USN, achieved fame as a physicist.

The detailed description of these measurements by Michelson[8] himself are available in most libraries and will not be repeated here. The distance used was 1986 feet and displacements as great as 133 millimeters were obtained. The light path was along what was then the north sea wall of the Naval Academy grounds. Since that date the grounds have been extended by filling in so that many of the academic buildings now stand on the path. The northwest end, where a frame building was constructed to house the measuring equipment, is now covered by the Marine Engineering buildings and the far end by the Natatorium. None of the original equipment remains at the Academy. Those pieces of apparatus from the physics laboratory were no doubt returned to their original use, while the special equipment was either Michelson's private property or was borrowed from other institutions. The one possible exception is the heliostat which was loaned by the Army Medical Museum of Washington, D. C. and is believed to be still in the possession of that institution.

In 1879 Michelson was promoted to Master, and in the fall of that year he was transferred to the Nautical Almanac Office in Washington. The director of this office, Professor Simon Newcomb, was at the time engaged in an attempt to repeat Foucault's measurements on a larger scale, and, no doubt, this was the reason for the transfer. It has been stated in this journal by Lemon[9] that Michelson worked under Newcomb while the latter was a professor at the U. S. Naval Academy. The records of the Naval Academy do not show that Simon Newcomb was ever a member of the faculty, nor does Newcomb make any mention of it in his rather complete autobiography.[10] Simon Newcomb was a commissioned officer in the now extinct Corps of Professors of the U. S. Navy during most of his professional life, but his duties were all at the Naval Observatory or the Nautical Almanac Office, both of which are in the nation's capital. The results of Michelson's work in collaboration with Newcomb can be found in the latter's final report[11] of his researches on the velocity of light.

In 1880 Master Michelson took a leave of absence from the Navy (not an uncommon procedure at the time) and went to Europe for advanced study. On September 30, 1881 he resigned his commission, having already been offered the chair of physics at the Case School of Applied Science.

During World War I, Professor Michelson was again on the Navy Register as a Lieutenant Commander, U. S. Naval Reserve. This tour of duty was spent in his own laboratory at the University of Chicago, where he developed a new optical range finder for use on naval vessels.

The often quoted[12] advice allegedly given by

[6] A. A. Michelson, *Am. J. Sci.* (3) **15**, 394 (1878); *Am. J. Sci.* (3) **17**, 324 (1879); *Nature* **18**, 195 (1878).
[7] *New York Daily Tribune*, May 17, 1879.
[8] A. A. Michelson, *Proc. AAAS* **28**, 124 (1879); *Am. J. Sci.* (3) **18**, 390 (1879); *Nature* 21, 94 (1879); *Astro. Papers, U. S. Nautical Almanac Office* **1**, 115 (1880).

[9] H. B. Lemon, *Am. Physics Teach.* **4**, 1 (1936).
[10] Simon Newcomb, *The reminiscences of an astronomer* (Houghton-Mifflin).
[11] Simon Newcomb, *Astr. Papers, U. S. Nautical Almanac Office* **2**, 235 (1891).
[12] *New York Times*, May 10, 1931, Sec. 1, p. 3, column 3.

Admiral Worden, then superintendent of the Naval Academy, to Midshipman Michelson to the effect that more time spent on gunnery and less on physical experiments would be to the benefit of both Michelson and the Navy certainly does not express the feeling of Navy men in general toward either Michelson or physical experiments. As an illustration in support of this contention, on May 8, 1948 a laboratory, built at a cost of $7,000,000 for "basic and applied research" in physics and related sciences, was dedicated as the Michelson Laboratory at the U. S. Naval Ordnance Test Station, China Lake, California. Dr. R. A. Millikan gave the "Michelson Memorial Address" on this occasion, and three of Professor Michelson's daughters were present as guests of the Navy. The original scroll of Professor Michelson's Nobel Prize, presented not long ago by one of his daughters, Mrs. Nevile Gardiner of Washington, D. C., is one of the most prized items in the U. S. Naval Academy Museum. On April 2, 1946 Congressman Carl Hinshaw of California introduced into Congress a bill (H.R. 5964) to provide for the erection at the Naval Academy of a suitable memorial to Albert Michelson. Having had no success in his first attempt, Congressman Hinshaw introduced a similar bill (H.R. 79) into the 80th Congress on January 3, 1947, but the bill was referred to the Committee on House Administration and never reported out.

In conclusion, I wish to make acknowledgment to Dr. William H. Crew and Professor Earl W. Thomson, whose companion articles[13, 14] on Michelson and the Velocity of Light first interested me in further study of the life and work of this great American physicist.

[13] W. H. Crew, *U. S. Naval Inst. Proc.* **56**, 38 (1930).
[14] E. W. Thomson, *U. S. Naval Inst. Proc.* **56**, 42 (1930).

Present Trends of University Courses in General Physics for Premedical Students*

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THE last quarter century has seen a great change in the attitude of both professors of physics and administrators of medical schools towards the problem of giving a course in physics suitable for students planning a career in medicine or in one of the biological or medical sciences. In the 1920–30 period the American Physical Society took a rather definite stand against special courses and, strangely enough, this was supported by the medical schools. All the considerations need not be reviewed but the practical difficulty of offering a separate section and the dominance of the engineers in the classes were among them. One would suspect also that the lack on the part of the physicists of appreciation of the basic importance of their science to medicine and their unfamiliarity with suitable illustrative material may have strongly influenced them. On the other hand, few medical men had received sufficient training or the sort of instruction in physics to make them fully conscious of the part physics might play in both the theory and the practice of their profession. The student, therefore, falling between the two groups, can hardly be blamed for his openly expressed resentment of the requirement that physics be taken and for his dislike of the subject.

The 1930–40 decade saw a decided change in these matters. The applications of physics and particularly of the techniques of physics to the problems of biology and of medicine expanded rapidly and in the laboratories of the biological sciences, particularly in physiology, the equipment came to resemble more and more the equipment of a physical laboratory. Electronic circuits came into general use—not merely in the radio field but also in all branches of research. They were particularly useful in the study of nerve action as well as in many forms of therapy. Radiology, both x-ray and gamma-ray, became increasingly reliable as physical controls, precision measurements, and so on, changed medical practice in this field from a so-called art to a scientific procedure. The physicist became a consultant—often a full-time member of the clinical staff having to do with cancer—and physicists as well as medical men became increasingly conscious of the part being played by physics in the science and practice of medicine.

* A condensed statement of the main points included in a paper given at the New York meeting of the AAPT, February 4, 1950.

In some colleges and universities extra lectures on applications were given those intending to enter medicine and courses in radiology became more scientific than empirical in nature. But the fundamental need was not met by any such adjustment. The writer became convinced that the basic training in physics could be given effectively only be weaving the applications of physics to the medical sciences into the very fabric of the general course in physics itself, just as has been the practice in courses in physics for engineers for many years. He therefore adopted the plan of supplying to his students notes on certain applications of physics to the medical sciences and of giving some indication of their importance *at the time these principles were being taught.* This plan has now been followed sufficiently long to obtain abundant and convincing evidence of its soundness. He has received assurance from medical schools that students do make freer use of their knowledge of physics if they are led to make the connection while they are still in the physics class. Many of his former students, now in medical practice, are actually making not only greater but more intelligent use of physical principles than is likely to be made by anyone who took his course in physics without realizing its value in medicine.

Experience over a considerable period would seem to support the following conclusions:

(1) The general course in physics for the premedical student must include applications of the subject in the field of his interests and these must be given in direct connection respectively with the principles being taught. They must be introduced as normal material, never forced. They should not be reserved for a section in fine print at the close of the chapter or for special chapters. If the instructor treats them as afterthoughts the students will look upon them the same way.

(2) Students will respond favorably to a course designed specifically to meet their needs. They are as likely to value physics as highly as any other of the basic sciences included in their course, even though the subject must continue to be a heavy one. The premedical student is likely to be superior to the average student, since the competition for entrance to medical schools is known to be severe. He is capable and willing to work provided he is led to realize, at the time he is called upon to do the work, that it is worth while.

(3) The premedical student who becomes convinced that physics is a basic science in his field will not only obtain a better grasp of the subject but will actually observe many more applications of it in his later work than will the student who, in conformance with tradition, has no sense of its value.

The 1940–50 decade has witnessed a rapid change in the attitude towards physics, not only of the public, in general, but of the students in the physics classes. It has been so repeatedly stated that the last war was, and that future wars will be physical, that the present generation of students have not only a respect for but a dynamic interest in the subject that did not characterize the student of a generation ago. There is no longer any doubt as to the basic nature of the science nor of its importance in *any* field of human endeavour, particularly in medicine. The influence of the younger medical men trained in the preceding decade, is being increasingly felt. The war veterans who saw so many phases of physics in action, particularly in the treatment of the wounded and in the physical therapy employed in their rehabilitation, have a wholesome respect for the subject. The new electrocardiographs and encephalographs, the electron microscopes, modern types of x-ray installations, are familiar tools of great value. The cyclotrons, betatrons, and other high power equipment have received much publicity. Radioactive tracers have opened entirely new approaches to many problems of basic nature in all the biological sciences. It would be as hard to find a medical faculty now which does not have high regard for physics as it would have been two decades ago to find one which would aggressively utilize physics or even insist that its students acquire a specialized and adequate knowledge of the subject.

Fortunately, it is becoming easier for the departments of physics to offer more suitable training to those interested in the medical sciences. The number of applications in modern textbooks of physics is definitely increasing.

Journals in physiology, in anatomy, and the medical texts and current literature now furnish more examples than at any previous time. The American Association of Physics Teachers has taken positive steps towards the collection of suitable illustrative material. Over 250 authors have collaborated under the editorship of Dr. Otto Glasser to produce the 1800-page compendium *Medical Physics* to which a second volume is now being added.* There are under preparation various textbooks designed to meet the needs of the nonengineering student as well as the traditional text meets the requirement of those going into engineering.

The responsibility for giving a suitable premedical course in physics is rapidly being shifted to the professors of physics, as material becomes increasingly available. Since most universities must divide their physics classes in any event, the matter of cost is hardly a major issue. The author is not advocating that the professor of physics should offer a course in biophysics, nor does he attempt to do so himself; but he does believe that his course should provide an adequate basis for such a course in much the same way courses in chemistry give the foundation for biochemistry. This is becoming increasingly important as the number of universities establishing departments of biophysics grow. He suggests no radical changes in the laboratory work since the present experiments do support the basic principles of physics, lead to the acquirement of useful techniques and to the acquaintance with instruments of wide use in the medical sciences and medicine. A more sympathetic attitude towards the interests of the premedical student shown in the distribution of emphasis among the topics presented and particularly in the selection of illustrative material, is all that is required. Experience has shown that such a course can be just as thorough and acceptable for credit in an arts college as the more traditional course. It can be made quite as attractive to the general student since he is likely to be far more interested in the applications of physics to his body than to engineering projects. Indeed, less than half of those now enrolling in the author's class expect to enter medicine.

The author would like to learn of the experience of others giving courses similar to that described above, and to receive suggestions as to applications which they have found suitable. Any found usable will be duly acknowledged in any material submitted for publication.

* Volume II is now available.

Wisconsin Section

The Wisconsin Section of the American Association of Physics Teachers had its annual meeting at Carroll College, Waukesha, Wisconsin on April 28 and 29, 1950. Meetings were held in the New Science Hall and luncheon Saturday in Voorhies Dormitory. Forty members attended the conference. Tours of the Waukesha Motor Works and of the Dairy Farms were provided for the guests. PRESIDENT N. V. RUSSELL, *Carroll College*, extended greetings to the guests on behalf of the host institution.

At the business meeting full consideration was given to the proposed changes in the constitution and by-laws of the AAPT. A program of invited and contributed papers was presented as follows:

Nature and objectives of the physics program at Carroll College. V. P. BATHA, *Carroll College.*

Two simple pieces of apparatus for the general physics course: a refractometer and a practice switchboard. J. BRADFORD, *Beloit College.*

Photomultiplier tubes as scintillation counters. T. SCOLMAN AND R. R. PALMER, *Beloit College.*

Electromagnetic radiation and weather activity. RALPH H. BETER, *Marquette University.*

An application of a simple method of measuring short-time intervals. FRANK G. KARIORIS, *Marquette University.*

Geiger counters, etc. J. L. DURANZ OR EDWARD REIBLE, *Nuclear Company.*

Magnetic pole strength *vs.* magnetic moment in teaching magnetism. J. G. WINANS, *University of Wisconsin.*

Age of the universe. A. E. WHITFORD, *University of Wisconsin.*

Science in general education. W. P. CLARK, *Eau Claire State Teachers College.*

The sound motion picture "Atomic Energy" was shown at the Friday evening session. A demonstration of new apparatus and devices was a feature of the meeting. Several papers which were presented by the authors at the Annual Meeting of the AAPT in New York were reviewed by members of the Section. These included:

The training of college physics teachers. CLAUDE E. BUXTON, *Yale University.* Review by E. V. BRIGGS, *Superior State Teachers College.*

A unified approach to college physics. NOEL C. LITTLE, *Bowdoin College.* Review by W. P. GILBERT, *Lawrence College.*

Officers elected at the meeting for the ensuing year were as follows; *President,* V. P. BATHA, *Carroll College; Vice-president,* J. R. DILLINGER, *University of Wisconsin; Secretary-Treasurer,* MISS MONICA BAINTER, *Stevens Point State Teachers College.*

W. P. CLARK, *Secretary*

Progress in Studies of the Airglow in Upper Air Research *

C. T. Elvey
U. S. Naval Ordnance Test Station, China Lake, California

TWO years ago at the meeting of this Society at The University of California at Los Angeles I reviewed the many methods of studying the upper atmosphere and in particular, discussed a program which was being supported by the Office of Naval Research at the U. S. Naval Ordnance Test Station. The object of the program is to learn as much of the physical state of the upper atmosphere as possible through spectroscopic and photometric observations of the light of the night sky, including the twilight, and I would like to present the progress we have made.

First, however, we will briefly review the nature of the light of the night sky. It is composite in origin and only a part of it is of interest in connection with the upper atmosphere. Various estimates have been made of the percentage contributions of the components to the total light of the night sky. However, these are necessarily difficult owing to some of the components being a continuous spectrum and others a bright-line spectrum. The sources of light of the night sky with continuous spectra are: the unresolved stars, galactic light, zodiacal light, and light from all astronomical sources scattered by the atmosphere. The sources of light with line spectra are the diffuse nebulae of the galaxy, the aurorae, and the light emitted by the upper atmosphere. The latter has been called the permanent aurora, the nonpolar aurora, and more recently, just "the light of the night sky." None of these names is particularly appropriate, especially the last; hence, I have recently adopted the term "airglow" to designate the light, other than the polar aurorae, emitted by the upper atmosphere. The airglow is apparently ever-present over the entire earth; it is difficult to disentangle it from the aurora at large magnetic latitudes since both are spectra of the air but under different conditions of excitation. The excitation of the aurora is considered to be through bombardment of the atmosphere by charged particles from the sun, while the airglow is probably caused by collisional processes in the upper atmosphere in which the energy of dissociation is released by recombinations exciting the atoms and molecules. Such processes will cause the emitted light to come from layers in the atmosphere owing to the negative exponential change of density with height.

The principal radiations in the airglow are the green and the red auroral lines of atomic oxygen, forbidden transitions; the D lines of sodium; certain bands of the first positive system of the nitrogen molecule; the forbidden Vegard-Kaplan system of the nitrogen molecule; and the Herzberg bands, also forbidden, of the oxygen molecule.

Instrumentation

During 1948 the program was devoted to the design and construction of the instruments, and to the development of facilities. In 1949, the techniques of observing and reducing the data have been established.

Two observing sites have been set up, one at Cactus Peak approximately 30 miles NNW of the Michelson Laboratory, and the other in the White Mountains NE of Bishop, California. The station at Cactus Peak is at elevation 5415 ft and the station in the White Mountains, called Blanc's Bluff is 10,500 ft. The line of sight distance of Blanc's Bluff from Cactus Peak is approximately 100 miles and the line bears 12°30′ west of north. Field stations on a cooperative basis have been set up in three other localities, one at the Haute Provence Observatory in Southern France, another at a proposed site for an observatory in the Belgian Congo, and the third near the Dartmouth College Observatory, Hanover, New Hampshire. Short field trips have been made to the University of Alaska at College, Alaska; Yerkes Observatory, Williams Bay, Wisconsin; and the McDonald Observatory, Fort Davis, Texas.

One spectrograph has been employed in the program to date. It is an instrument especially

* Invited paper presented at the Stanford meeting of the American Physical Society, December 29–30, 1949.

constructed for Raman spectroscopy, and primarily for use in a laboratory. Its principal disadvantage is lack of portability. The instrument was built by Huet of Paris and employs a large prism of extra dense flint glass as the dispersive system. The camera lens is a modified and enlarged microscope objective with a numerical aperture $F/0.7$. The resulting linear dispersions are: 150 A/mm at 4000A; 345 A/mm at 5000A; 640 A/mm at 6000A; and 1080 A/mm at 7000A. Owing to the dense flint optics the spectrograph cannot be used for wave-lengths shorter than 4100A. In the red region of the spectrum the low dispersion is a disadvantage. Owing to the good definition, however, reasonably accurate wave-lengths have been measured.

It was decided that the photometric part of the program should receive the major attention. Since the program called for the measurement of the intensities of the main radiations of the airglow as functions of zenith distance in various directions in the sky throughout the night, and also as functions of time, it was decided to construct a recording photometer. The photometer was designed and described by Douglas Marlow and J. C. Pemberton.[1] It uses a 1P21 phototube as the sensitive element. An $F/1$ lens, 4-in. diameter, forms an image of the sky on a rectangular aperture thus admitting an area $2°\times 10°$ to the sensitive surface. The long edge of the aperture is parallel to the horizon. A 30-cycle chopper in front of the phototube produces a pulsed photoelectric current which is amplified with a gain of 5×10^{6} in the 1P21 and an additional gain of 1.3×10^{5} in a two-stage, negative-feedback amplifier of high stability and linearity. The over-all gain is 6.5×10^{11}. Synchronous rectification is used and the output is recorded with an Esterline-Angus recording milliammeter. The minimum energy which can be recorded reliably is approximately 10^{-13} watt.

The photometer is automatic with the controls at the recorder. A continuous patrol may be kept of any part of the sky, or the photometer can be made to sweep the sky from horizon to horizon along any vertical circle, or may be made to survey the sky beginning on the northern horizon sweeping the meridian to the south point, rotating in azimuth $22\frac{1}{2}°$ and sweeping the sky again until eight sweeps are made. At the end of the eighth sweep the instrument automatically returns to the starting point and begins another survey. A survey of the sky is accomplished in 32 minutes.

At the present time, five such instruments are in operation, or are being set up for operation in the near future. Two are at Cactus Peak, one near Hanover, New Hampshire, one in southern France, and the other in the Belgian Congo.

Spectroscopy

Spectroscopic studies of the light emitted from the upper atmosphere, either the aurora or the airglow, will give us considerable knowledge concerning the kinds of atoms, molecules and ions present, their modes of excitation, and other physical data from which it will be possible to deduce the chemical and physical state of the upper atmosphere as a function of time.

Our program did not anticipate a study of the auroral spectrum. However, we did send a field trip to College, Alaska and there obtained some excellent auroral spectra for studies in connection with the airglow spectrum. Since our spectrograph gives rather low dispersion but has great light gathering power, most of the effort on the field trip was devoted to the faint upper parts of auroral rays, faint diffuse auroras, and faint patches, rather than to the bright parts.

A comparison of the spectrum of a bright auroral ray, exposure 20^{m} with that of the faint upper parts of the rays, exposure $8^{h}25^{m}$, is shown in Fig. 1. Although the exposures were such as to give approximately the same densities for the more prominent features of the auroral spectrum, many of the faint features are quite different. This is, no doubt, an altitude effect in which many of the emissions in the aurora change relative intensity. For example, the line at 5200A which has been identified by several investigators as a forbidden nitrogen line, ${}^{4}S-{}^{2}D$, may be considered. Although this line is similar in origin to the transition in the oxygen atom that produces the red auroral line, it is very much fainter, and appears at higher altitudes. De-excitation by collisions is no doubt a very important matter in determining its intensity since the lifetime in the excited state is approxi-

[1] *Rev. Sci. Inst.* **20**, 724 (1949).

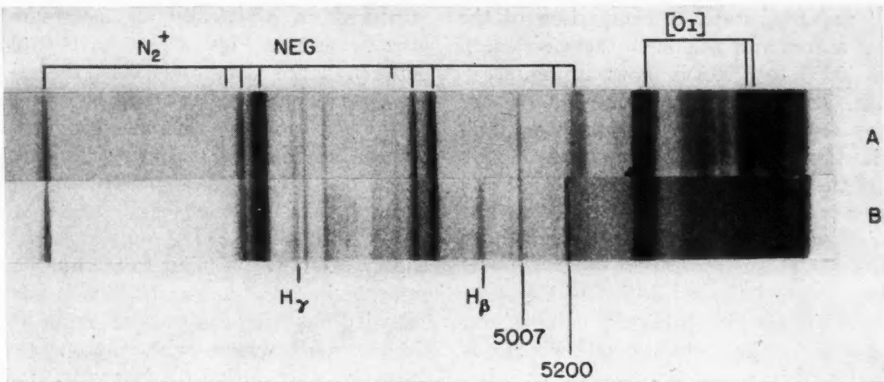

FIG. 1. Comparison of the spectrum of a bright auroral ray (A) with that of the faint upper parts of the ray (B).

mately eight hours. The lifetimes in the excited states for the green and red auroral lines of oxygen are of the order of 0.4 sec and 100 sec, respectively.

Of special note are the hydrogen lines which are broad and, as has been pointed out by other investigators,[2] indicate large random velocities of the hydrogen atoms. The random velocities are somewhat difficult to explain, but may be a result of the release of atoms from molecules rather than incoming hydrogen atoms from the sun. The first three members of the Balmer series are present in auroral spectra; H_β is the only unblended line. Line H_α is rather mixed up with the bands of the first positive system of nitrogen, although in spectra of a very faint aurora or airglow using low dispersion, it is possible to distinguish its presence when the bands are not too intense. On the spectrograms of aurora taken with the *CI* spectrograph when no hydrogen is present, six distinct members of the first positive system are seen. When hydrogen is present, as evidenced by H_β, the two bands of the first positive system, one on each side of the position H_α, are blended into a very wide structure and enhanced relative to the other four bands, thus indicating the presence of H_α.

The spectrum of the airglow differs radically from that of the aurora in most spectral regions. In the near ultraviolet portion of the auroral spectrum the dominant features are bands of the second positive group of nitrogen, corresponding to the transition, $C^3\pi_u \rightarrow B^3\pi_g$, with excitation potential 10.96 v, while in the airglow they are exceptionally weak, if present. Most of the stronger features of the airglow spectrum in this region have been attributed to the Herzberg bands of oxygen, arising from the forbidden transition $^3\Sigma_u{}^+ \rightarrow X^3\Sigma$ with excitation potential 4.5 v. The blue-violet region of the auroral spectrum is dominated by the bands of the first negative system of the ionized nitrogen molecule's transition, $A'^2\Sigma \rightarrow X'^2\Sigma$, with an excitation potential 18.7 v above the ground level of the neutral molecule. The same region of the airglow spectrum does not exhibit these bands but does show faintly a group of wide bands which are identified as the Vegard-Kaplan bands of nitrogen due to the forbidden transition $A^3\Sigma_u \rightarrow X'\Sigma_g$, with excitation potential 6.2 v. In the visual region of the spectrum the differences are less marked. The green and red auroral lines, forbidden transitions in the oxygen atom, $^1D-^1S$, and $^3P-^1D$, with excitation potentials of 4.2 v and 1.96 v, respectively, are the prominent features in both the spectra. In addition, the auroral spectrum exhibits strong bands of the first positive system of nitrogen in the red region, whereas in the airglow their appearance is uncertain. On the other hand, in the infrared region, the (0, 0) band of the first positive system of nitrogen, if correctly identified, is by far the strongest emission in the airglow spectrum. It shows that molecules producing the airglow spectrum can be excited only to the lowest

[2] C. W. Gartlein, *Physical Rev.* **74**, 1208 (1948).

levels of the $B^3\pi_g$ state. A comparison of the spectra of aurora and airglow in the wavelength region of 4000A to 7000A is shown in Fig. 2.

Both spectra in the region 7000–10,000A are rich in emission features, but many are still unidentified. A. B. Meinel[3] has discussed this portion of the spectrum.

Figure 3 is a composite of three spectrograms of the airglow taken with three different emulsions using the *CI* spectrograph.

We have studied the red region of the spectrum of the airglow with particular emphasis on the band of uncertain origin at 6560A which was announced by Elvey and Farnsworth[4] as being enhanced in the twilight and which has been considered to be a blend of bands of the first positive system of nitrogen. It is now apparent that such a band does not exist. The tentative identification, now unsubstantiated, is attributed to the low linear dispersion used, the spectral sensitivity of the emulsion, the atmospheric absorption bands, and the reddening of the twilight at low altitudes.

A new line, mean wavelength 6847A, has been measured on practically all spectrograms. As may be seen in Figs. 2 and 3, its intensity is between that of 6300A and 6364A, the red auroral lines. Its identity has not been established.

A study of some 30 spectrograms of the aurora and the airglow have given strong evidence that hydrogen is present in the upper atmosphere.[5] The wavelength of the emission agrees with that of H_α, and in addition, its appearance is very similar to the hydrogen lines observed in auroras. The H_α line is variable in intensity but has been present on all 15 of the airglow spectrograms with exposures of four hours or longer during the last year. Horace W. Babcock,[6] using a grating spectrograph with higher dispersion, recorded a line of the same description at that wavelength. Since hydrogen lines appear in high altitude auroras and since all of our spectrograms were taken of the sky toward the north and mostly at a zenith distance of 80°, it is possible, although it does not seem probable, that we have observed distant auroras on each occasion. Under these conditions auroras at 1000 km would have been in the

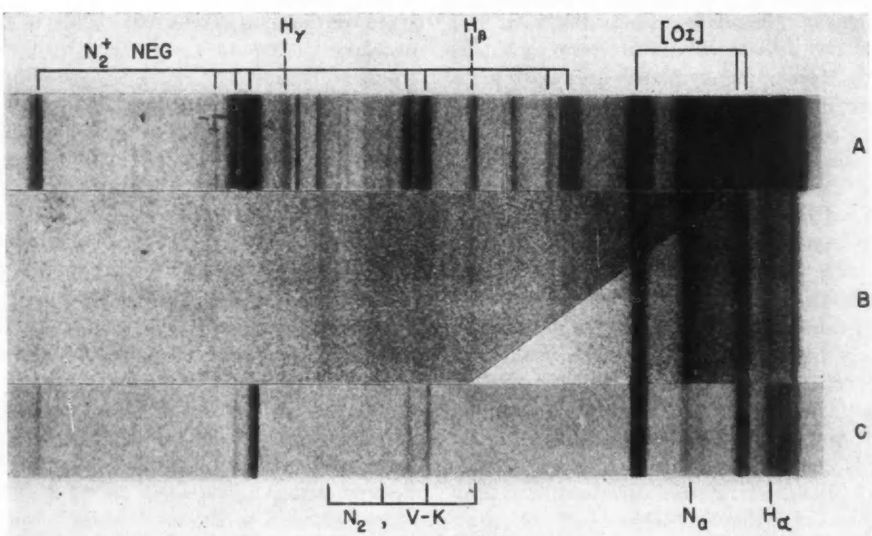

FIG. 2. Comparison of the spectrum of the airglow (*B*) with that of the upper parts of auroral rays (*A*) and with diffuse aurora (*C*).

[3] In a private communication Meinel has recently analyzed the band structures in this region of the airglow spectrum and the resulting molecular constants indicate a hydride, probably OH.
[4] *Astrophysical J.* 96, 451 (1942).
[5] Meeting of the American Astronomical Society at Tucson, December 28–31 (1949).
[6] *Publ. Astro. Soc. Pacif.* 51, 47 (1939).

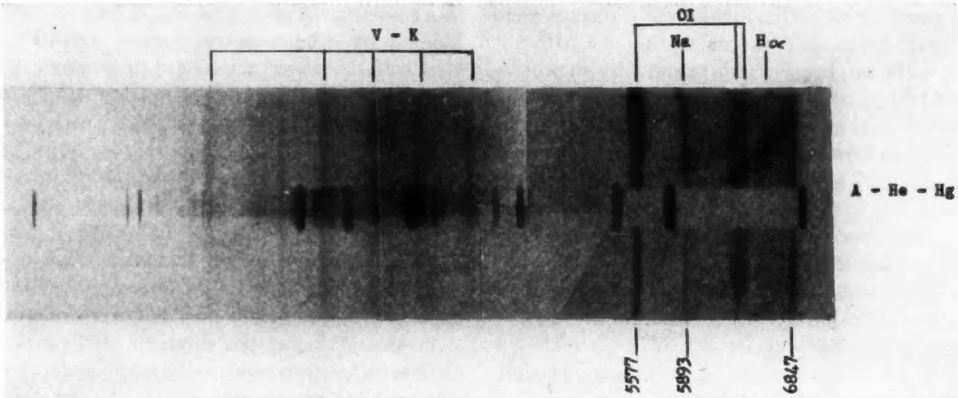

FIG. 3. Spectrum of the airglow with a comparison spectrum of argon, helium and mercury.

zenith well above the Canadian border, or auroras at 500 km would have been at the zenith in northern United States. Since Babcock observed the sky at zenith distance 50° from the Lick Observatory, he should also have detected such auroras. (During one of his exposures he did detect a faint aurora which lasted a short time.) A crude estimate of the intensity of H_α indicates less than 10^6 quanta per sec per cm^2 column at the zenith.[7]

Photometry

The principal problem in the photometric program is a study of the stronger emission lines of the airglow spectrum with the aim of determining the height in the atmosphere at which the radiations are produced. This is important for two reasons; first, the excitation is probably by collisions, and second, both the green and red auroral lines of oxygen are transitions from metastable states, one with a lifetime of 0.4 sec, the other with a lifetime of about 100 sec, and de-excitation by collisions will occur. These processes of exchange of energy are dependent upon the density of the particles in the atmosphere and their velocities.

Two methods of determining the heights of the emitting layers were considered feasible, one known as the van Rhijn method and the other a triangulation method in which two or more stations would try to observe the same irregularity in the emitting layer.

The theory of the van Rhijn method[8] of determining the height to an emitting layer in the upper atmosphere gives the ratio of intensity at a given zenith distance to that at the zenith as a function of the height of the emitting layer. The ratio may be written in the form

$$\frac{I_{(z)}}{I_{(0)}}=\left\{1-\left(\frac{R}{R+h}\right)^2\sin^2 z\right\}^{-\frac{1}{2}}e^{-\tau m}S_c, \qquad (1)$$

where $I_{(z)}$ is the observed intensity at zenith distance z, $I_{(0)}$ the observed intensity at zenith, R the radius of the earth, h the height of layer (thin) above surface, $e^{-\tau m}$ the extinction factor, and S_c the scattering factor.

The basic assumptions in this theory are: (1) the layer is thin, and (2), it is uniform over large geographical areas, which further implies that it is of uniform density and height.

In practice it is customary to compute a family of curves using Eq. (1) for assumed values of the height to the layer. A family of such curves is shown in Fig. 4, and one set of data, represented by black spots, is included.

It soon became evident that the assumption of uniformity over large geographical areas was

[7] Recent observations of the airglow in the southern sky shows no evidence of H_α. Furthermore, through the kindness of Professor W. Petrie of the University of Saskatchewan we find that virtually all of the 15 airglow spectograms noted in the text were obtained during periods of auroral activity. It thus seems highly probable that the H_α observed must be excited by auroral activity. Further observations are being made, taking simultaneous observations of the northern and southern skies.

[8] *Pub. Astr. Lab. Groningen*, No. 31, (1921).

not justified as radically different results were obtained from different azimuths. An attempt was made to choose uniform data by making a polar plot of the intensity for given zenith distances as a function of azimuth. If the assumption of uniformity is correct the plots should give circles; hence, in this manner attempts were made to pick areas of the sky free from distortions and then compare the observations with the van Rhijn formula.

A frequency plot of observed heights obtained by Roach and Barbier[9] of the sodium line 5893A and the forbidden oxygen line 5577A are shown in Fig. 5. At the present time, it appears that the sodium line of the airglow originates at a height between 250 and 300 km and the green auroral line at approximately 90–100 km. These determinations by Roach and Barbier are definitely superior to those obtained by Elvey and Farnsworth[4] and by A. and E. Vassey[10] since conditions of observation which more nearly fit the assumptions of the van Rhijn theory were selected, and more precise corrections for extinction and scattering have been applied.

During the past summer some excellent sets of data were obtained by Dr. Roach and coworkers, for use in the triangulation method of determining the heights to the irregularities or patches of light emitted by the atmosphere. Complete surveys of the sky and continuous sweeps along a common vertical circle were obtained at the two field stations, Cactus Peak and White Mountain, for the green oxygen line.

The boundaries of the patches in the emitting

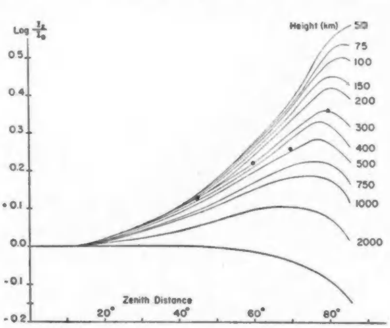

FIG. 4. Curves representing Eq. (1) for various values of the height *h*.

[9] *Publ. Astro. Soc. Pacif.* **61**, 88 (1949).
[10] *Physical Society Gassiot Committee Report*, p. 53, (1948).

layer were such as to prevent determinations of heights by trigonometric means. Detailed analyses of the observations are progressing, however, and it appears that the presently accepted height to the green oxygen layer will be confirmed.

One very interesting result of the photometric studies of the sodium emission was made by Barbier and Roach,[11] in which they found an excess of sodium light in the west following the end of twilight and in the morning before the beginning of dawn. The position of the maximum of intensity was at the azimuth of the sun and its intensity decreased as the sun's depression increased. It appears that this is a resonance phenomenon, as is the twilight enhancement of the sodium observed spectrographically, and that the emitters are the sunlit sodium atoms in a column outward from the edge of the earth's shadow in the atmosphere. Computing the height to the earth's shadow, the variation of intensity as a function of height is obtained. Furthermore, these intensities may be translated into densities of sodium atoms, and it is found that the distribution of sodium atoms in the region 200–600 km indicates a scale height of 250 km. These data indicate either: (1) that the distribution of the molecules is contrary to normal expectations, or (2) that sodium atoms are not representative of the atmosphere as a whole, a conclusion that would imply an extraterrestrial origin for the sodium atoms in the upper atmosphere.

The absolute intensities of the green auroral line and the sodium line have been measured using the stars for calibration. The average intensity of the green auroral line is 5×10^8 quanta/cm^2 sec; the nocturnal variation has been well established, the intensity increasing approximately twofold to a maximum which is reached sometime between 2200 and 0200 local time, and then decreasing to an intensity at dawn near that at the beginning of the night. The curve of intensity-variation is indicative of a growth and decay phenomenon.

The average intensity of the sodium line is about 8×10^7 quanta/cm^2 sec, but the most striking thing about the sodium is the annual variation. There is a fivefold variation of intensity between the minimum in the late summer

[11] *Publ. Astro. Soc. Pacif.* **61**, 91 (1949).

and the maximum during the winter. Since the source of sodium may be from the faint meteors evaporating in the upper atmosphere, this may explain part of the annual variation. However, account must be taken of the absorption due to water vapor since one of the absorption bands partly overlaps the sodium emission.

Our photometers are well suited for studies of the scattered sunlight at the zenith during the twilight period. Observations have been made from sunset until the end of astronomical twilight and several good series of data have been recorded. Direct scattering theory can be used to represent the observations to the end of civil twilight, that is, till the zenith distance of the sun is 96°, but beyond this point multiple scattering rapidly becomes important. At the present time this portion of the program is being critically evaluated to see if a further expenditure of time is profitable without the introduction of extensive laboratory investigations on the scattering process.

Briefly summarizing, the spectroscopic observations of faint high altitude auroral rays and diffuse aurorae show some very interesting facts which have not been completely analyzed. The H_α line has been identified as a faint component of the airglow spectrum; it may, however, be a result of distant high altitude aurorae. The photometric observations show that the green auroral line of oxygen has an intensity of 5×10^8 quanta/cm^2 sec and is emitted in a region near the E-layer of the ionosphere at maximum in the winter; that the sodium D-lines correspond to 8×10^7 quanta/cm^2 sec, show a large annual variation, and are emitted from the general vicinity of the night F-layer. Furthermore, resonance excitation of sodium atoms at twilight can be detected to heights of 800 km in the atmosphere.

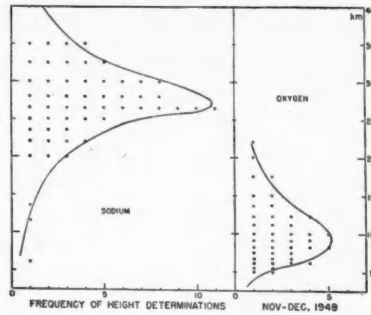

FIG. 5. Frequency of height determinations for sodium (5893A) and oxygen (5577A) during observing periods in November and December 1948.

Since many of the above conclusions are still tentative, the future program will thus continue along the same lines. Particular effort will be made to verify the identification of H_α in the airglow spectrum and determine if it is present at low magnetic latitudes. Also, absolute intensities of the fainter radiations will be made from the spectrophotometric observations, basing them on the accurate observations made of the stronger radiations with the photoelectric photometer. The heights to the emissions of the green auroral line, and the sodium lines will be placed on a firmer basis and efforts will be made to study the upper atmospheric motions from these data.

This work has been carried out under Project No. NR-082-045 of the Office of Naval Research, and besides the writer, the following persons have taken part in the investigation: E. V. Ashburn, Daniel Barbier, Institut d'Astrophysique, Paris, John B. Irwin, University of Indiana, Douglas Marlow, J. C. Pemberton, Helen Pettit, F. E. Roach, Armin Wiebke, and Don Williams. Dr. P. Swings of University of Liége, Belgium, has served as consultant during the two years' duration of the project.

Fundamentals of Nuclear Magnetic Resonance Absorption. I*†

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Washington University, Saint Louis, Missouri

Contents of Part I*

A. Introduction

B. Magnetic and Angular Momentum Properties of the Nucleus
1. The Nuclear Magnet and Its Vector Model
2. The Larmor Precession; Energy in the Magnetic Field
3. The Nuclear Magneton and "The Magnetic Moment"
4. Thermal Agitation: The Curie Susceptibility

C. Magnetic Resonance
5. Absorption and the Bohr Frequency Condition
6. The Flopping Torque of a Precessing Magnetic Field
7. The Quantum-Mechanical Transition Probability
8. The Spin-Lattice Relaxation Time, T_1
9. Spin-Spin Interactions

D. The Bloch Formulation
10. Absorption and the Complex Susceptibility
11. The Bloch Equations
12. The Bloch Susceptibilities

Appendix

List of Principal Symbols

A. Introduction

THE magnetic properties of the nucleus have interested the physicist ever since they were first postulated to explain the hyperfine structure of spectral lines. It was supposed that the nucleus is, in general, a small magnet whose interaction with the atomic electrons splits the energy levels between which the electrons make the transitions responsible for atomic line spectra. With the development of spectroscopic equipment of high resolving power, it ultimately became possible to determine certain nuclear spins and to measure a number of nuclear magnetic moments to about two significant figures.[1]

Considerably more accurate measurements were obtained in molecular beam experiments,[2,3] which were pioneered under O. Stern and which later flourished under Rabi at Columbia University. An outstanding addition to beam technique was the magnetic resonance method[2,3] which the Rabi group applied to these experiments. The most recent developments in the study of nuclear magnetic moments have applied the magnetic resonance principle to solids, liquids, and gases in their normal physical states, and the compilation of data concerning nuclear magnetism has become even more rapid.

The new resonance techniques, devised simultaneously and independently by the Purcell and Pound group[4,5] at Harvard and the Bloch and Hansen group[6,7] at Stanford, require relatively simpler equipment than do the beam experiments. (Indeed, a bridge-type apparatus similar to that of Bloembergen, Purcell, and Pound is part of an outstanding undergraduate program in physics directed by Professor Frank Verbrugge at Carleton College.) Further, the new methods seldom require the alteration of the physical or chemical form of the sample containing the nucleus whose magnetic properties are to be studied.

In addition to the extremely important advances in measurement of nuclear magnetic properties which the new methods provide, they also afford means for investigating establishment of the thermal equilibrium essential to magnetic methods for attaining very low temperatures. And the width and shape of the resonance has yielded information concerning crystal structure,[8,9] phase transitions in solids,[10] and hindered internal motions in solids.[11] An excellent intro-

* The concluding portion of this article will appear in a later issue of this Journal.

† Assisted by the joint program of the ONR and AEC.

1 H. Kopfermann, *Kernmomente* (Edwards Brothers, Inc., Ann Arbor, 1945), Chapter II.

2 D. R. Hamilton, *Am. J. Physics* **9**, 319 (1941).

3 J. B. M. Kellogg and S. Millman, *Rev. Mod. Physics* **18**, 323 (1946).

4 Purcell, Torrey, and Pound, *Physical Rev.* **69**, 37 (1946).

5 Bloembergen, Purcell, and Pound, *Physical Rev.* **73**, 679 (1948), referred to as BPP throughout the following pages.

6 Bloch, Hansen, and Packard, *Physical Rev.* **69**, 127 (1946).

7 F. Bloch, *Physical Rev.* **70**, 460 (1946); Bloch, Hansen, and Packard, *Physical Rev.* **70**, 474 (1946).

8 G. E. Pake, *J. Chem. Physics* **16**, 327 (1948).

9 Gutowsky, Kistiakowsky, Pake, and Purcell, *J. Chem. Physics* **17**, 972 (1949).

10 N. L. Alpert, *Physical Rev.* **75**, 398 (1949).

11 H. Gutowsky and G. E. Pake, *J. Chem. Physics* **18**, 162 (1950).

ductory paper by Purcell,[12] titled "Nuclear magnetism in relation to problems of the liquid and solid states," skillfully presents certain potentialities of nuclear magnetism in structural studies to the reader who is without previous background in the subject.

One should not infer from the foregoing that molecular beams are in any sense outmoded. The very effects which reflect the structure of the sample tend, for many substances, to render the new techniques less effective in measuring the asymmetry of the nuclear charge distribution (the electric quadrupole moment). Nor have the Purcell and Bloch experiments offered competition to their contemporary cousin, microwave spectroscopy,[13] or to molecular beams in the determination of nuclear spins. Instead of seeking to eliminate one another, these three experimental approaches—molecular beams, microwave spectroscopy, and nuclear resonance absorption (Purcell) or nuclear resonance induction (Bloch)—supplement each other to provide versatile means for probing phenomena involving the magnetic and electric moments of nuclei.

The present article attempts to discuss one of these new methods, the nuclear resonance absorption technique of Purcell and Pound, in such a way as to be of value to the first-year graduate student of physics, and the material is substantially that used in introducing nuclear magnetism to graduate students at Washington University, where research begins with the inception of graduate study. A knowledge of the Bohr frequency condition is assumed, but no familiarity with quantum mechanics is required. A few results are taken bodily from quantum mechanics and are accompanied by remarks which, it is hoped, will put the student on the alert in his future studies for transition probabilities, the uncertainty principle, and other paraphernalia of quantum mechanics.

The closely allied nuclear induction experiment is not treated here, inasmuch as the pioneering articles on this subject by Bloch and his collaborators[7] provide in one place a lucid and sufficiently comprehensive treatment. These papers are quite as valuable to the beginning graduate student as to the front line researcher, for Bloch takes full advantage of the validity of the classical equations of motion in determining the nuclear magnetization. Lack of familiarity with quantum mechanics is thus much less of a stumbling block for the student when he reads these papers than when he turns to existent literature on nuclear absorption.

For the expert physicist who has specialized in other branches of his subject and wishes to become acquainted with the short history of this new nuclear technique, it is hoped that the present article can at least serve as a partially complete guide to the literature.

B. Magnetic and Angular Momentum Properties of the Nucleus

1. The Nuclear Magnet and its Vector Model

In addition to its well-known properties of mass, charge, and intrinsic angular momentum (spin), the atomic nucleus possesses in general a magnetic moment, that is, it behaves much as if it were a bar magnet. Although elementary textbooks often define the magnetic moment of a magnet as the product of its magnetic pole strength and the distance between its two poles, physicists believe that free magnetic poles do not exist and that a more accurate picture involves circulating electric currents or "current whirls." The magnetism of a bar of iron is attributed to "current whirls" of a kind within the iron atoms, and similarly the magnetism of the nucleus may be considered as originating with circulating currents in the nucleus. Since the existence of an intrinsic angular momentum of the nucleus implies a circulation of mass within it, it should not be surprising that the magnetic moment and angular momentum are related to each other. Indeed, classical models of the nucleus predict that these two vectors should be collinear and that their lengths should always be in the same ratio. For example, if one computes the magnetic moment of a spinning spherical shell with charge q and mass M uniformly distributed over its surface, he obtains a magnetic moment

$$\boldsymbol{\mu}=\frac{q}{2Mc}\mathbf{p}, \tag{1.01}$$

[12] E. M. Purcell, *Science* **107**, 433 (1948).

[13] See the review article by W. Gordy, *Rev. Mod. Physics* **20**, 668 (1948).

TABLE I. Nuclear spins, nuclear magnetic moments in units of the nuclear magneton, and the nuclear resonant frequency for $H_0 = 10{,}000$ gauss. The third column is taken from Taub and Kusch, *Physical Rev.* **75**, 1481 (1949); for the limits of error of these numbers, see this reference. The last column does not utilize all the significant figures of the fifth column, but instead indicates to one decimal place the magnitude of the resonant frequency in megacycles $\sec^{-1}$. Nuclear magnetic moments obtained from hyperfine structure are generally known to about two significant figures, and are not included in this table. In addition, no exception has yet been found to the empirical rule that nuclei with even A and Z have zero spin and zero magnetic moment.

Z	Nucleus	A	Spin	$\mu = gI$	$\nu_0 = g\mu_0 h^{-1} H_0$ for $H_0 = 10^4$ gauss (megacycles/sec)
0	*n*	1	1/2	−1.9135	29.1
1	H	1	1/2	2.7935	42.6
1	H	2	1	0.8576	6.5
1	H	3	1/2	2.9797	45.4
2	He	3	1/2	2.13	32.5
3	Li	6	1	0.8223	6.3
3	Li	7	3/2	3.2571	16.5
4	Be	9	3/2	−1.177	6.0
5	B	10	3	1.8012	4.6
5	B	11	3/2	2.6893	13.7
6	C	13	1/2	0.7025	10.7
7	N	14	1	0.404	3.1
7	N	15	1/2	±0.280	4.3
9	F	19	1/2	2.6291	40.1
11	Na	22	3	1.7464	4.4
11	Na	23	3/2	2.2178	11.3
13	Al	27	5/2	3.6419	11.1
15	P	31	1/2	1.1318	17.3
17	Cl	35	3/2	0.8222	4.2
17	Cl	37	3/2	0.683	3.5
19	K	39	3/2	0.391	2.0
19	K	40	4	−1.291	2.4
19	K	41	3/2	0.215	1.1
29	Cu	63	3/2	2.2266	11.3
29	Cu	65	3/2	2.3850	12.1
31	Ga	69	3/2	2.0145	10.3
31	Ga	71	3/2	2.559	13.0
38	Br	79	3/2	2.1061	10.7
38	Br	81	3/2	2.2700	11.5
37	Rb	85	5/2	1.3534	4.1
37	Rb	87	3/2	2.7510	14.0
49	In	113	9/2	5.489	9.3
49	In	115	9/2	5.502	9.3
53	I	127	5/2	2.8105	8.6
55	Cs	133	9/2	3.316	5.5
56	Ba	135	3/2	0.8364	4.3
56	Ba	137	3/2	0.9354	4.8
81	Tl	203	1/2	1.6121	24.6
81	Tl	205	1/2	1.6280	24.8

where $\mathbf{p}$ is the angular momentum of the spinning shell and c is a constant numerically equal to the velocity of light in free space.

We shall see that the nucleus does not conform accurately to this model nor to any other proposed simple model (nor even, thus far, to any complex one!), and that the resonance absorption phenomenon to be discussed here provides one of the experimental methods for learning the relation between the nuclear magnetic moment and the vector $\mathbf{p}$. However, nuclear magnets differ only in magnitude and/or sign from the prediction of Eq. (1.01), and it is customary to write

$$\boldsymbol{\mu} = g\frac{e}{2Mc}\mathbf{p}, \tag{1.02}$$

where g, called the nuclear g factor ~~gyromagnetic ratio~~, is a number characteristic of a given nuclear species in a given nuclear energy state. The symbol M denotes the proton mass and e the proton charge.

Since the nuclear moment is proportional to the intrinsic angular momentum or *spin* of the nucleus, it is worth while to review the special properties of this vector according to modern physics. The angular momentum of any particle or system of particles is found to be easily expressible in terms of a fundamental unit $\hbar$, which is Planck's constant divided by 2π. The quantity usually denoted "the spin" is defined as $1/\hbar$ times the largest observable value of the time average of a component of $\mathbf{p}$ in a given direction. We shall usually be concerned with magnetic fields and we select the direction of an applied magnetic field $\mathbf{H}$ as the direction of interest to us:

$$\text{"the spin"} = (1/\hbar)(p_H)_{\max} = I. \tag{1.03}$$

It is found experimentally and theoretically that the nuclear spin I can have integral or half-integral values. Each nuclear ground state is characterized by just one value of I, which is tabulated under the column marked "Spin" in Table I.

A general expression for all the permitted values of p_H is

$$p_H = m\hbar, \tag{1.04}$$

where $m = I, I-1, I-2, \cdots, -I+1, -I$. Since quantum-mechanical arguments show that the value of $\mathbf{p}\cdot\mathbf{p}$ is $I(I+1)\hbar^2$, the length of the angular momentum vector is

$$|\mathbf{p}| = [I(I+1)]^{\frac{1}{2}}\hbar. \tag{1.05}$$

One can apply these relations to find the length of the component at right angles to the $\mathbf{H}$-direction. Consider the proton, which has been found experimentally to have $I = \frac{1}{2}$. When $p_H = m\hbar$, with $m = +\frac{1}{2}$, the length of the vector is $|\mathbf{p}| = (\sqrt{3}/2)\hbar$ and the perpendicular component

is

$$p_\perp = (\sqrt{2}/2)\hbar, \tag{1.06}$$

which is even greater than $(p_H)_{max}$. As I increases, $(p_H)_{max}$ and $|\mathbf{p}|$ become more nearly equal. For an object which is massive (compared to a nucleus), such as a spinning baseball, $(p_H)_{max}$ and $|\mathbf{p}|$ are to all intents and purposes equal for measurable values of $\mathbf{p}$.

2. The Larmor Precession; Energy in the Magnetic Field

If a magnet of dipole moment $\boldsymbol{\mu}$ is placed in a magnetic field $\mathbf{H}$, a torque is exerted on the magnetic dipole,

$$\mathbf{L} = \boldsymbol{\mu} \times \mathbf{H}. \tag{2.01}$$

Newton's law for rotational motion states that the rate of change of angular momentum of a system is equal to the torque applied to it, or

$$d\mathbf{p}/dt = \mathbf{L}. \tag{2.02}$$

Since the torque on a nucleus with magnetic moment $\boldsymbol{\mu}$ is given by Eq. (2.01), it follows that

$$d\mathbf{p}/dt = \boldsymbol{\mu} \times \mathbf{H}. \tag{2.03}$$

But since $\boldsymbol{\mu} = g(e/2Mc)\mathbf{p}$, we have

$$d\mathbf{p}/dt = -g(e/2Mc)\mathbf{H} \times \mathbf{p}, \tag{2.04}$$

which is the equation of motion for a vector $\mathbf{p}$ of constant magnitude precessing with angular velocity

$$\boldsymbol{\omega}_0 = -g(e/2Mc)\mathbf{H}. \tag{2.05}$$

To see this, note that (Fig. 1)

$$d\mathbf{p} = \boldsymbol{\omega}_0 \times \mathbf{p}dt \tag{2.06}$$

gives the proper direction to $d\mathbf{p}$, as well as the proper magnitude.

We therefore conclude that if a nucleus of magnetic moment $\boldsymbol{\mu} = g(e/2Mc)\mathbf{p}$ is placed in a magnetic field, the magnetic moment vector (or the angular momentum vector) precesses with the angular frequency [Eq. (2.06)] regardless of the angle between $\boldsymbol{\mu}$ and $\mathbf{H}$. This is called the *Larmor precession frequency*.

Recalling the arguments of Sec. 1, we see that the vector model of the nucleus is easily described in terms of the magnetic moment as follows. The nuclear magnetic moment of a nucleus with spin I is a vector of length

$$|\boldsymbol{\mu}| = g(e/2Mc)[I(I+1)]^{\frac{1}{2}}\hbar; \tag{2.07}$$

it has a component

$$\mu_H = g(e\hbar/2Mc)m \quad [m = I, I-1, \cdots, -I] \tag{2.08}$$

along the direction of an externally applied magnetic field $\mathbf{H}$, and a component of length

$$\mu_\perp = g(e\hbar/2Mc)[I(I+1) - m^2]^{\frac{1}{2}} \tag{2.09}$$

which is perpendicular to the external field and precesses with an angular frequency of magnitude

$$\omega_0 = g(e/2Mc)H. \tag{2.10}$$

Since, as may be derived from Eq. (2.01), the potential energy U of a magnetic moment $\boldsymbol{\mu}$ in a magnetic field $\mathbf{H}$ is, apart from an additive constant,

$$U = -\boldsymbol{\mu} \cdot \mathbf{H} = -\mu_H H, \tag{2.11}$$

the energy of our nuclear dipole in a state characterized by m is

$$U(m) = -g(e\hbar/2Mc)mH, \tag{2.12}$$

and a nucleus of spin I has, in general, $2I+1$ energy levels (one for each value of m) accessible to it in consequence of its interaction with a magnetic field $\mathbf{H}$. These are called the Zeeman levels, inasmuch as they are similar to those responsible for the Zeeman splittings in atomic

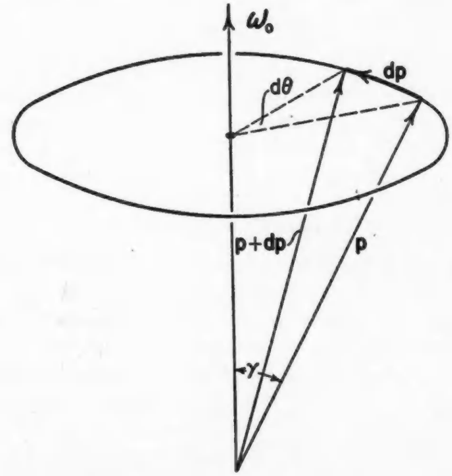

FIG. 1. Vector diagram used to illustrate the differential equation obeyed by a precessing angular momentum vector of constant magnitude.

spectra. It should be noted explicitly that the foregoing equations are valid whether g is negative or positive.

3. *The Nuclear Magneton and "The Magnetic Moment"*

The constant

$$\mu_0 = eh/2Mc \tag{3.01}$$

is called the nuclear magneton, and nuclear magnetic moments are often measured in terms of it. The *Bohr magneton*, which is the unit of measure for electronic magnetic moments, may be found from Eq. (3.01) by substituting the electron mass for the proton mass. Note that electronic magnetic moments are of the order of 1000 times larger than nuclear moments. The values of the two magnetons are as follows:

$$\mu_0 = 5.049 \times 10^{-24} \text{ erg/gauss}$$
$$\mu_\beta = 0.9273 \times 10^{-20} \text{ erg/gauss.}$$

It is customary to let the vector $\mathbf{I}$ stand for the nuclear spin in units of $\hbar$. Then

$$\mathbf{I}\cdot\mathbf{I} = I(I+1)$$
$$I_H = m.$$

We can combine these definitions to express most conveniently the nuclear magnetic moment vector as

$$\boldsymbol{\mu} = g\mu_0\mathbf{I}. \tag{3.02}$$

The quantity colloquially referred to as "the magnetic moment" is

$$(\mu_H)_{\max} = gI\mu_0. \tag{3.03}$$

The dimensionless number gI is the "magnetic moment" measured in units of the nuclear magneton; it is the number tabulated in the fifth column of Table I.

4. *Thermal Agitation: The Curie Susceptibility*

We introduce the effects of thermal agitation on an assembly of magnets by carrying out a simple derivation of the so-called static Curie susceptibility.

The magnetic induction, or magnetic flux density, within a sample of matter may be written as

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}, \tag{4.01}$$

where $\mathbf{H}$ is the magnetic field and $\mathbf{M}$ is the volume density of magnetic dipole moments or the magnetic moment per unit volume of the sample. In simple substances which are isotropic, the vector $\mathbf{M}$ is proportional to the applied magnetic field, as we shall find in our ensuing computation, and the proportionality factor χ is defined as the magnetic susceptibility:

$$\mathbf{M} = \chi\mathbf{H}. \tag{4.02}$$

Consider now a substance containing N nuclei per unit volume, and assume for simplicity that their nuclear magnets are all alike. Then we can compute the magnetic moment per unit volume, $\mathbf{M}$, if we know the number of nuclei in each energy state, since we already know from Eq. (3.08) the value of the static magnetic moment component along the field direction for each state m. The natural tendency is for a magnet to align itself parallel to an external field, for, as shown by Eq. (2.12), the potential energy is then a minimum. However, the energy possessed by each degree of freedom of the nucleus by virtue of its temperature is $\frac{1}{2}kT$, and a simple calculation shows that, even for the largest nuclear magnet in Table I, this thermal energy far exceeds the difference in energy between the parallel and antiparallel positions of the nuclear magnet in the field $\mathbf{H}$.

We therefore expect the collisions due to thermal agitation in the sample to play such havoc among the ranks of the nuclear magnets, which the external magnetic field $\mathbf{H}$ seeks to align, that there will be but a very small excess of nuclei in the lowest energy state. To illustrate this effect quantitatively, we compute from the Boltzmann factor the excess number of protons $(2I+1=2)$ in the lower state at room temperature in a field of 20,000 gauss (usually the upper limit on fields obtainable in the laboratory).

Let the number of nuclei per unit volume with energy $U(m)$ be $N(m)$. Then the ratio of the population of the two proton states $m=\frac{1}{2}$ and $m=-\frac{1}{2}$ is, according to Boltzmann,

$$\frac{N(+\frac{1}{2})}{N(-\frac{1}{2})} = \frac{\exp[-U(\frac{1}{2})/kT]}{\exp[-U(-\frac{1}{2})/kT]} \cong 1 + g\mu_0\frac{H}{kT}, \tag{4.03}$$

since, at room temperature, $U(m) \ll kT$. Putting the numbers $H = 20{,}000$ gauss, $kT = 4\times10^{-14}$ erg, and $g = 5.58$ for protons, one finds

$$N(+\tfrac{1}{2})/N(-\tfrac{1}{2}) = 1 + 14.1\times10^{-6}.$$

Thus, for every million nuclei in the upper energy state, there are one million and fourteen nuclei in the lower energy state. Clearly it is these fourteen protons in each two million which are responsible for the net nuclear magnetization of the sample.

To compute the Curie susceptibility for any nucleus characterized by $\alpha=g\mu_0H/kT$, we require the sum

$$M_H=\sum_{m=-I}^{I} N(m)\mu_H(m) =K\sum_{m=-I}^{I}\exp[m\alpha]g\mu_0 m. \qquad (4.04)$$

Since $\alpha \ll 1$, the constant K is easily evaluated from

$$N=\sum_{-I}^{I} N(m)=\sum_{-I}^{I} K\exp[m\alpha]$$

as $N/(2I+1)$. Thus Eq. (4.04) becomes

$$M_H=\frac{N}{2I+1}\sum_{-I}^{I}(1+\alpha m)\alpha\frac{kT}{H}m$$

$$M_H=\frac{N}{2I+1}\alpha^2\frac{kT}{H}\sum_{-I}^{I}m^2. \qquad (4.05)$$

Whether I is integral or half-integral, it may be shown that

$$\sum_{-I}^{I} m^2=\tfrac{1}{3}I(I+1)(2I+1),$$

and the final result is

$$M_H=(N/3kT)g^2\mu_0^2I(I+1)H. \qquad (4.06)$$

Comparison with Eq. (4.02) identifies the Curie susceptibility, which we denote by χ_0:

$$\chi_0=(N/3kT)g^2\mu_0^2I(I+1). \qquad (4.07)$$

Returning to our example, the proton, we see that Eq. (4.07) simply gives the magnetic moment per unit volume arising from the fourteen excess protons aligned parallel to $\mathbf{H}$ out of every two million protons in the sample. With reduced temperatures, the excess population in the lower energy state increases. Indeed, at very low temperatures the nuclear effect becomes so pronounced as to be experimentally measurable; the static Curie susceptibility of solid hydrogen has been measured directly by Lasarew and Schubnikow.[14]

C. Magnetic Resonance

5. Absorption and the Bohr Frequency Condition

On the basis of the Bohr frequency condition, one can use the results of Part B to conclude that nuclear resonance absorption may occur. Bohr's explanation of the hydrogen spectrum involved the postulate that a system characterized by two discrete energy states separated by energy ΔU may make a transition from one state to the other accompanied by either emission or absorption of a quantum of electromagnetic radiation of energy

$$\hbar\omega=\Delta U. \qquad (5.01)$$

Whether the quantum is emitted or absorbed is determined by energy conservation for the transition in question.

In Part B a nucleus in a magnetic field was found to have $2I+1$ energy levels accessible to it, and if $I\neq 0$, transitions are possible. The energy difference between any two such levels in a constant external magnetic field $\mathbf{H}_0$ is

$$U(m'')-U(m')=g\mu_0H_0(m'-m''). \qquad (5.02)$$

Only transitions in which m changes by $+1$ or -1 are permitted by a so-called *selection rule* and therefore transitions are permitted between adjacent states of an energy level scheme such as that of Fig. 2, which applies to a nucleus with $I=5/2$. The selection rule applied to Eqs. (5.01) and (5.02) determines the frequency of the radiation emitted or absorbed by the nuclear magnetic dipole:

$$\hbar\omega_0=g\mu_0H_0, \qquad (5.03)$$

which is precisely the Larmor frequency of Eq. (2.10). Protons in a field of 10,000 gauss precess at a frequency $\omega_0/2\pi=42.6\times10^6$ sec^{-1}, which is in the radiofrequency range.

To summarize, if one subjects a sample containing nuclear magnets to radiation at the Larmor frequency, which is the order of megacycles in ordinary laboratory magnetic fields, a nucleus in a lower Zeeman energy state may ab-

[14] B. Lasarew and L. Schubnikow, *Physik. Zeits. Sowjetunion* 11, 445 (1937). If this reference is not available, see reference 1, pp. 224–225.

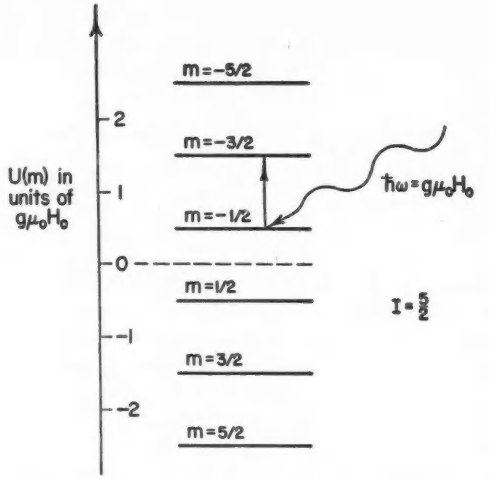

FIG. 2. Energy level diagram for a nuclear moment of spin 5/2, showing schematically the absorption of a quantum of radiation which induces a transition between a pair of adjacent Zeeman levels.

sorb a quantum of energy from the radiation field and make a transition to the next higher energy state. If the frequency of the radiation is not near the Larmor frequency, we expect little or no absorption, and hence the absorption is what physicists call a *resonance phenomenon.*

This cursory argument ignores entirely a number of important details. For example, are there sufficient excesses of nuclei in the lower energy states for a measurable net absorption to occur? If so, what means exist to maintain this distribution during the absorption process? And, are there any details about the radiation, such as its polarization, which are important? All of these questions will be dealt with in later sections.

6. The Flopping Torque of a Precessing Magnetic Field

Suppose that a small magnetic field $\mathbf{H}_1$ rotating with angular frequency ω is in some way placed at right angles to the constant magnetic field $\mathbf{H}_0$, where $H_1 \ll H_0$. The additional field $\mathbf{H}_1$ will produce a new torque $\mathbf{L}_{\text{flop}}$ which tends to tip the vector $\boldsymbol{\mu}$. Note, however, that if $\mathbf{H}_1$ rotates at a frequency appreciably different from the Larmor precession frequency of $\boldsymbol{\mu}$ in the large magnetic field, $\mathbf{L}_{\text{flop}}$ will change sense periodically with a frequency which is the difference between ω and ω_0. This is seen from Fig. 3 by noting that when $\mathbf{H}_1$ has the position OA with respect to $\boldsymbol{\mu}$, $\mathbf{L}_{\text{flop}}$ tends to tip $\boldsymbol{\mu}$ downward, but when $\mathbf{H}_1$ has position OB, its torque ($\mathbf{L}'_{\text{flop}}$) tends to tip $\boldsymbol{\mu}$ upward. This periodic change in sense of $\mathbf{L}_{\text{flop}}$ when ω and ω_0 differ appreciably leads to a zero time average for the flopping torque.

But when $\omega = \omega_0$, $\mathbf{L}_{\text{flop}}$ tends always to tip $\boldsymbol{\mu}$ in the same sense. If the vector $\boldsymbol{\mu}$ does tip and thus change its angle with respect to $\mathbf{H}_0$, energy must either be absorbed or emitted; again we are led to the resonance condition, this time by purely classical considerations.

The magnetic field $\mathbf{H}_1$ must *rotate* with angular frequency ω_0, whereas it seems natural to think of the electromagnetic radiation mentioned in Sec. 5 as possessing an *oscillatory* magnetic component. Indeed, the two points of view are reconcilable if one notes that an oscillatory field consists of two superimposed fields which rotate in opposite directions. Consider the field

$$\left.\begin{aligned} H_x &= 2H_1 \cos\omega t \\ H_y &= 0 \\ H_z &= 0 \end{aligned}\right\}. \tag{6.01}$$

This is evidently expressible as the sum of the two fields,

$$\left\{\begin{aligned} H_x^{\text{right}} &= H_1 \cos\omega t \\ H_y^{\text{right}} &= H_1 \sin\omega t \\ H_z^{\text{right}} &= 0 \end{aligned}\right\}\left\{\begin{aligned} H_x^{\text{left}} &= H_1 \cos\omega t \\ H_y^{\text{left}} &= -H_1 \sin\omega t \\ H_z^{\text{left}} &= 0 \end{aligned}\right\}, \tag{6.02}$$

which rotate about the z axis with frequency ω, but in opposite directions. If the oscillating field Eq. (6.01) has $\omega = \omega_0$, one of its rotating components will follow the precessing vector $\boldsymbol{\mu}$ and produce transitions as discussed in this section. The oppositely rotating component is far off resonance, and its effect is negligible.

These considerations lead us to expect that the radiation which is to induce the transitions of Sec. 5 should be polarized with its magnetic vector at right angles to the large field $\mathbf{H}_0$.

7. The Quantum-Mechanical Transition Probability

The simple arguments of Secs. 5 and 6 attempt to make the absorption plausible, but a quantum-mechanical treatment is required to establish firmly the properties of the absorption by an

assembly of nuclear magnets. Quantum mechanics does not give us complete information as to the energy, angular momentum, and position of each nucleus at any time, but it does provide us with all that we need to know, namely, the *probability* that a nuclear magnetic moment initially in a state m will at some later time t be found in a state m'. This probability, expressed *per unit time*, will be denoted $P(m\rightarrow m')$.

If a nucleus in one of its Zeeman energy states is immersed in a radiation bath with energy in the frequency range $d\nu$ near ν given by $\rho(\nu)d\nu$, one expects the probability of a transition to be proportional to the number of quanta present with frequency near the Larmor frequency, that is, proportional to $\rho(\nu_0)$. In fact the quantum-mechanical result obtained by perturbation theory is

$$P(m\rightarrow m')=(2\pi/3\hbar^2)g^2\mu_0^2|\mathbf{I}_{mm'}|^2\rho(\nu_0). \quad (7.01)$$

The quantity $|\mathbf{I}_{mm'}|$, which is the so-called *matrix element* of the nuclear spin, is usually of order of magnitude unity; when $|m'-m|>1$ it vanishes, giving rise to the selection rule mentioned in Sec. 5.

For an isolated magnetic moment with $I=\frac{1}{2}$ in a constant magnetic field $\mathbf{H}_0$ perpendicular to which is a precessing smaller field $\mathbf{H}_1$, Rabi[15] computes the transition probability directly from the Schrödinger equation containing the time, without resorting to perturbation theory. He finds that the chance $C(\frac{1}{2}\rightarrow -\frac{1}{2})$ for a nucleus initially in state $m=\frac{1}{2}$ to be at a later time t in state $m=-\frac{1}{2}$ is

$$C(\tfrac{1}{2}\rightarrow -\tfrac{1}{2})=\frac{\sin^2\theta}{1+(\omega_0/\omega)^2-2(\omega_0/\omega)\cos\theta}\times\sin^2\left\{\frac{\omega t}{2}\left[1+\left(\frac{\omega_0}{\omega}\right)^2-2\frac{\omega_0}{\omega}\cos\theta\right]^{\frac{1}{2}}\right\}, \quad (7.02)$$

where $\tan\theta=H_1/H_0$; note that C is *not* a probability per unit time, but is rather the total probability at any time t. The ratio ω_0/ω is to be counted negative if the rotation of $\mathbf{H}_1$ is not in the same sense as the nuclear precession. In most experiments, $H_1\ll H_0$ and Eq. (7.02) becomes

$$C(\tfrac{1}{2}\rightarrow -\tfrac{1}{2})=\frac{\theta^2}{[1-(\omega_0/\omega)]^2+(\omega_0/\omega)\theta^2}\times\sin^2\left\{\frac{\omega t}{2}\left[\left(1-\frac{\omega_0}{\omega}\right)^2+\frac{\omega_0}{\omega}\theta^2\right]^{\frac{1}{2}}\right\}. \quad (7.03)$$

When resonance is obtained, that is, when $\omega=\omega_0$, a value of t can be found for which $C(\frac{1}{2}\rightarrow -\frac{1}{2})$ is as close to unity as one pleases. Moreover, when $\omega=-\omega_0$, $C(\frac{1}{2}\rightarrow -\frac{1}{2})$ becomes extremely small, confirming the conclusion of Sec. 7 that the wrong-rotating half of the oscillating field has negligible effect.

For higher values of the spin I, Majorana[16] has obtained the general formula for $C(m\rightarrow m')$.

At resonance, the expression (7.03) oscillates between 0 and 1 as time progresses. This hints at a result which is explicitly found by Rabi and also by the perturbation treatment, namely,

$$\begin{aligned} C(\tfrac{1}{2}\rightarrow -\tfrac{1}{2})&=C(-\tfrac{1}{2}\rightarrow \tfrac{1}{2})\\ P(\tfrac{1}{2}\rightarrow -\tfrac{1}{2})&=P(-\tfrac{1}{2}\rightarrow \tfrac{1}{2}). \end{aligned} \quad (7.04)$$

The classical discussion made this seem likely, for there is nothing about the rotating field $\mathbf{H}_1$ which will turn off $\mathbf{L}_{\text{flop}}$ after the magnetic moment $\boldsymbol{\mu}$ has been tipped to a new position with respect to $\mathbf{H}_0$ (and, therefore, to a new energy).

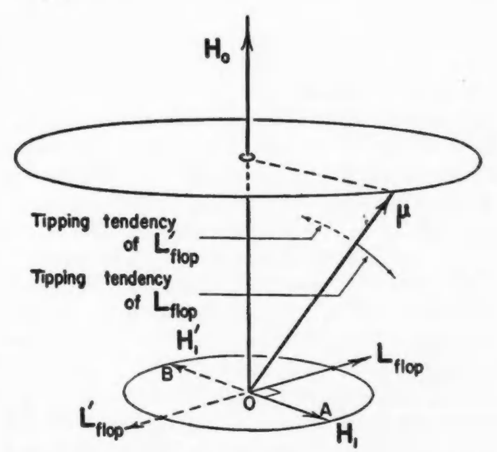

FIG. 3. Vector diagram illustrating the tendency of the small precessing magnetic field $\mathbf{H}_1$ to tip the magnetic moment vector as it precesses in a large constant field $\mathbf{H}_0$.

[15] I. I. Rabi, *Physical Rev.* **51**, 652 (1937). See also J. Schwinger, *Physical Rev.* **51**, 648 (1937).

[16] E. Majorana, *Nuovo Cimento* **9**, 43 (1932). See also reference 3.

The reader is cautioned to use discretion in applying conclusions drawn from Eq. (7.03) to an assembly of *interacting* spins. First, as will be seen later, $C(\frac{1}{2}\rightarrow-\frac{1}{2})$ becomes quite small if ω_0/ω differs from unity by even a very small amount, and inhomogeneities in any laboratory magnetic field will make it impossible for a sizable fraction of the nuclei initially in state $m=\frac{1}{2}$ to have their C's near unity at any instant. Further, the nuclear magnets produce time varying local magnetic fields at their neighbors which, tend to spread ω_0 over an appreciable range only a small portion of which will be covered at any instant by a single monochromatic radiation source.

The equal probability found in Eq. (7.04) for transitions downward in energy (stimulated emission) and transitions upward in energy (absorption) emphasizes that, if a net absorption is to occur, there must at all times be the excess number of nuclear magnets which we found in Sec. 4 to exist in the lower energy states at thermal equilibrium. Otherwise, every absorptive transition will be balanced, on the average, by an emissive transition, and there will be no net energy exchange. We are thus brought to grips with the problem: how is thermal equilibrium among spin states established, and, once established, can it be maintained during a resonance experiment? The answer is found in the spin interactions to be discussed presently.

8. *The Spin-Lattice Relaxation Time*, T_1

Consider a sample containing nuclear magnetic moments which resides initially in a small magnetic field such as that of the earth. The Boltzmann factors for the various Zeeman energy states are almost exactly unity and the spins are essentially equally distributed among the $2I+1$ spin states. If this sample is thrust suddenly into the gap of an electromagnet which produces a field of 10,000 gauss, we ask how much time must elapse before the equilibrium excess numbers of nuclei will have found their way into the lower energy states. We are thus interested in the mechanism of the so-called *relaxation process* by which equilibrium is established. From a thermodynamic point of view, a relaxation process is thus any method of energy exchange between the system of nuclear spins and the lattice.

At the first instant after insertion into the magnetic field, and before equilibrium attains, the nuclear spins are still very nearly distributed in equal numbers among the $2I+1$ levels. A glance at Eq. (4.03) indicates that, since H_0, and therefore $U(m)$, is now large, the exponent can remain negligible only if the temperature T_S of the spin system is extremely high. In fact, for $I=\frac{1}{2}$, the excess number in the lower energy state defines at any time the spin temperature T_S. Our problem is that of describing the heat exchange between the lattice of vibrating atoms or molecules and the system of nuclear magnets, whose existence is usually well-sheltered by the enveloping electron closed-shells formed during the chemical combination which brought the lattice into being.

In order for the nuclear spins to "cool down," it is necessary that transitions from the upper spin states to the lower spin states occur more frequently than the reverse transitions. This seems at first thought to be incompatible with the result of Eq. (7.03) that the probabilities for transitions in both directions are equal. However, the present situation differs from that considered in Sec. 7 in that the entire system, consisting of lattice+radiation field+spins, is being left to itself to come to equilibrium at a definite temperature, whereas the monochromatic radiation field of Sec. 7 never comes to equilibrium with the nuclear spins at a common single temperature. Therefore the probabilities of Sec. 7 cannot be applied to the relaxation process without first taking into account certain properties and consequences of the equilibrium.

Let $N(p)$ and $N(q)$ be the equilibrium populations of two Zeeman levels p and q which differ in energy by U_p-U_q. Then a detailed balancing of the transitions between p and q will preserve equilibrium if

$$N(p)W(p\rightarrow q)=N(q)W(q\rightarrow p), \qquad (8.01)$$

where $W(p\rightarrow q)$ is the total probability per unit time of a single transition from p to q. But, at equilibrium, the Boltzmann factor governs $N(p)/N(q)$, and

$$\frac{W(p\rightarrow q)}{W(q\rightarrow p)}=\frac{N(q)}{N(p)}=\exp\left(\frac{U_p-U_q}{kT}\right). \qquad (8.02)$$

Now the probability of a single transition from

p to q cannot depend upon the population of q, and we must suppose that the total probabilities W are related to the quantum-mechanical probabilities P through the Boltzmann factor of the final state, even when equilibrium has not yet obtained:

$$W(p\to q)=P(p\to q)\exp(-U_q/kT). \qquad (8.03)$$

We consider in detail the case $I=\frac{1}{2}$. Let the total number of spins be N, the population of the lower and upper energy states being respectively $N(+)$ and $N(-)$. Then, since $P(+\to-)=P(-\to+)=P$, we obtain

$$\begin{aligned} W(+\to-)&=P\exp(-\tfrac{1}{2}g\mu_0H_0/kT)\\ W(-\to+)&=P\exp(+\tfrac{1}{2}g\mu_0H_0/kT). \end{aligned} \qquad (8.04)$$

The excess number $n=N(+)-N(-)$ changes by 2 for each transition. This fact and the definition of the probabilities leads to the differential equation

$$dn/dt=2N(-)W(-\to+)-2N(+)W(+\to-). \qquad (8.05)$$

Since $n\ll N$ for all cases of interest here, we substitute

$$\begin{aligned} W(+\to-)&=P[1-\tfrac{1}{2}|g|\mu_0H_0/kT]\\ W(-\to+)&=P[1+\tfrac{1}{2}|g|\mu_0H_0/kT], \end{aligned} \qquad (8.06)$$

obtained from Eq. (8.04), into Eq. (8.06) to find

$$dn/dt=2P(n_0-n), \qquad (8.07)$$

where $n_0=N|g|\mu_0H_0/kT$ is the equilibrium value of n. Integration of Eq. (8.07) yields

$$n=n_0[1-\exp(-2Pt)]. \qquad (8.08)$$

We note that the equilibrium excess number establishes in a fashion analogous to the charging of a capacitor of time constant $1/2P$. This characteristic time,

$$T_1=1/2P, \qquad (8.09)$$

is called the *spin-lattice relaxation time* or the thermal relaxation time, and is the time required for all but $1/e$ of the equilibrium excess number to reach the lower energy state.

Theories of nuclear magnetic relaxation must compute P from Eq. (7.01). Waller[17] made one of the pioneering attempts in this direction. He examined the effect of the spectrum of vibrations and other lattice motions responsible for the specific heat of a solid. Such vibrations of the charged particles of the lattice lead to oscillatory currents and therefore to oscillatory local magnetic fields, the spectral intensity of which at the Larmor frequency should determine P according to Eq. (7.01). However, this mechanism leads to values of T_1 which are several orders of magnitude too large. Bloembergen, Purcell, and Pound[5] have studied relaxation in fluids and find that Brownian motions at the Larmor frequency provide the relaxation mechanism.

Thermal relaxation times thus far measured range from 10^{-4} sec or less in certain solutions containing paramagnetic ions to several hours for very pure ice crystals at liquid nitrogen temperatures. The value of T_1 for the protons in water at room temperature has been measured by Hahn[18] to be 2.33 ± 0.07 sec, in good agreement with the theory of BPP (see footnote 5). Relaxation for nuclear moments in solids, inadequately accounted for by Waller's theory, now appears to be explained by the presence of minute amounts of paramagnetic impurities, with their thousand-times larger magnetic moments.[19,20] These effects will be discussed in Part G.

9. Spin-Spin Interactions

The preceding section dealt with interactions between the system of nuclear spins and the lattice. In addition, each individual precessing nuclear moment interacts with neighboring spins through their magnetic fields. In fact, the total magnetic field at any single nucleus consists not only of the applied field H_0, but includes also the resultant of the local fields produced by the static components of neighboring magnetic dipoles. Depending upon the arrangement of its neighbors among the $2I+1$ values of μ_H, a given nucleus "sees" a slightly larger or slightly smaller field than that externally applied. One can estimate this effect by finding the magnitude of the local field which a nuclear magnetic dipole may be expected to produce at a distance of an angstrom unit or so,

$$H_{\text{loc}}\sim\mu_0r^{-3}\sim5 \text{ gauss}. \qquad (9.01)$$

[17] I. Waller, *Zeits. f. Physik* **79**, 370 (1932).

[18] E. L. Hahn, *Physical Rev.* **76**, 145 (1949).

[19] J. F. Darby and B. V. Rollin, *Nature* **164**, 66 (1949).

[20] N. Bloembergen, *Physica* **15**, 386 (1949).

We can expect a dispersion or spread of values of the precession frequency because of the variation over these several gauss of the effective magnetic field at different nuclei throughout the sample. The magnitude of the precession frequency spread, $\delta\omega_0$, is then

$$\delta\omega_0 \sim g\mu_0\hbar^{-1}H_{\text{loc}} \sim 10^4 \text{ sec}^{-1}. \qquad (9.02)$$

One can interpret this as meaning that, if two nuclei are known to be precessing in phase at time $t=0$, they may be expected to have lost their phase relationship within a time the order of $1/\delta\omega_0 \sim 10^{-4}$ sec.

A second process can occur to interrupt the phases of the precessing spins. If nuclei A and B are antiparallel to each other, the precessing component of A's magnetic moment produces at B a precessing magnetic field at nearly the proper frequency to produce a transition, and vice versa. It is, therefore, possible for A and B to flip each other over, leaving the net energy of the spin system unchanged. In effect, spins A and B have interchanged positions in the lattice, and this is often called a spin exchange or a spin-spin collision.

Though the spin-spin collision does not affect the total energy of the spin system, it does limit the lifetime of a spin state and leads, through the Heisenberg uncertainty relation, to an energy spread or dispersion. Since the relative phases of the two neighboring spins change appreciably during a time $1/\delta\omega_0$ (see the first part of this section), we can expect that nucleus A will require a time of that order of magnitude before its Larmor frequency becomes precisely equal to that of spin B, and the lifetime of a spin state should be limited by spin-spin collisions to times the order of $1/\delta\omega_0$. Then the energy levels are broadened by an amount δU given by

$$\delta U \cdot (1/\delta\omega_0) \sim \hbar \qquad (9.03)$$

in accordance with the Heisenberg uncertainty principle. We again arrive at the conclusion that the relative phases of the precessing nuclear spins will be destroyed over times the order of $1/\delta\omega_0 \sim 10^{-4}$ sec, as a consequence of the interruption of a precessing state through spin-spin collisions.

The observable effect of these two phase-destroying processes arises through the spread in energy levels, that is, the dispersion of Larmor frequencies, which imparts to the absorption line a finite width. Later sections will be devoted to a detailed discussion of line width and its meaning in terms of the internal structure of the sample.

The quantum-mechanical calculation of line width verifies that both local field dispersion and spin exchange are real effects which broaden the line. The uniqueness of the two may be established by noting that unlike nuclear neighbors, by virtue of their appreciably different precession rates, should be unable to participate in spin-exchange processes, whereas their static components along $\mathbf{H}_0$ are still perfectly capable of dispersing the values of the total field at nuclei in the sample. Theory and experiment verify this fact.

D. The Bloch Formulation

10. Absorption and the Complex Susceptibility

Our point of view thus far has often been *microscopic*, that is, we have considered various phenomena in terms of the individual nucleus. In an actual experiment, of course, one deals with matter in bulk, and it is the *macroscopic* magnetic moment of the sample as a whole which he observes. The magnetic moment per unit volume of the sample $\mathbf{M}$ is related to the magnetic field $\mathbf{H}$ through the magnetic susceptibility χ, which is conveniently selected as the quantity to which the absorption will be related.

As in the familiar example of the magnetic absorption which leads to hysteresis losses in a transformer core, the energy absorbed by unit volume of the sample per second is

$$A = \frac{\omega}{2\pi}\int_{t=0}^{t=2\pi/\omega} \mathbf{H}\cdot d\mathbf{M}. \qquad (10.01)$$

The integral itself represents the energy absorbed per cycle. It is convenient to represent the oscillatory magnetic field as the real part of a complex number

$$\mathcal{H} = 2H_1 \exp(i\omega t) \qquad (10.02)$$

(script letters are used throughout to denote complex quantities). Then the physically observable magnetization is the real part of $\mathcal{M}$ which is the product of the complex susceptibility

$\chi = \chi' - i\chi''$ and $\mathcal{H}$:

$$M = \chi'(2H_1 \cos\omega t) + \chi''(2H_1 \sin\omega t). \quad (10.03)$$

Since $2H_1 \cos\omega t$ is the applied field, the imaginary portion of the susceptibility is a measure of the out-of-phase component of magnetization.

If we take $\mathbf{M}$ and $d\mathbf{M}/dt$ to be collinear with $2\mathbf{H}_1$, the integral of Eq. (10.01) is quickly evaluated to yield

$$A = 2H_1^2\omega\chi''. \quad (10.04)$$

The energy absorbed by the nuclear spin system is therefore proportional to χ'', the out-of-phase component of the nuclear magnetization. In the following section the resonance properties of χ'' will be demonstrated, thereby establishing *nuclear magnetic resonance absorption*.

11. The Bloch Equations

The equations of motion involving the nuclear magnetization $\mathbf{M}$ and the total magnetic field can be written from the discussions of earlier sections. These equations were first set up and solved by F. Bloch,[7] who sought a phenomenological description of the nuclear induction effect which is closely related to the absorption treated here.

Let the sample be placed in a large constant magnetic field $\mathbf{H}_0$ which lies along the z axis of a rectangular coordinate system, and suppose a small magnetic field to precess about the z axis. Then the magnetic field vector is taken to include $\mathbf{H}_0$ and the "left" rotating portion of Eq. (6.01):

$$\left.\begin{aligned} H_x &= H_1 \cos\omega t \\ H_y &= -H_1 \sin\omega t \\ H_z &= H_0 \end{aligned}\right\}. \quad (11.01)$$

The rotating field has its angular velocity vector along the direction of $-z$ because Eq. (2.05) indicates that a positive magnetic moment precesses in this sense. The equations which follow therefore treat absorption by an aggregate of nuclei with positive magnetic moments; the results will be equally valid for a negative moment if the $\mathbf{H}_1$ field precesses about $+z$, but no distinction will be necessary in the experimental situation which uses the oscillating field $H_x = 2H_1 \cos\omega t$, as both precessing components are present.

It is assumed for simplicity that the sample contains but one magnetic nuclear species, of which there are N per unit volume, and that this nucleus has spin I and magnetic moment gI nuclear magnetons. Although overlapping of two different nuclear resonances will seldom occur because the resonances are generally quite narrow, superposition will adapt the results which follow to any situation.

The vector equation of motion for a single nuclear magnetic moment may be written from Eq. (2.03) as

$$d\boldsymbol{\mu}/dt = g(e/2Mc)\boldsymbol{\mu}\times\mathbf{H}. \quad (11.02)$$

Summing over all the nuclear moments in unit volume of the sample transforms Eq. (11.02) into

$$(d\mathbf{M}/dt) = \gamma\mathbf{M}\times\mathbf{H}, \quad (11.03)$$

where

$$\gamma = ge/2Mc = \omega_0/H_0. \quad (11.04)$$

The parentheses around $(d\mathbf{M}/dt)$ are intended to indicate that Eq. (11.03) represents but one of several contributions to $d\mathbf{M}/dt$, others arising from the processes discussed in Secs. 8 and 9. For example, the z-component of magnetization is proportional to the excess number in the lower energy states, and Eqs. (8.07) and (8.09) produce a spin-lattice contribution

$$(dM_z/dt) = (M_0 - M_z)/T_1. \quad (11.05)$$

The final contribution arises through spin-spin processes, which were found in Sec. 9 to destroy phase relationships between the precessing ($x-$ and $y-$) components of the nuclear magnetic moments. This destruction of the $x-$ and $y-$ components may be expressed by the equations

$$\begin{aligned} (dM_x/dt) &= -M_x/T_2 \\ (dM_y/dt) &= -M_y/T_2. \end{aligned} \quad (11.06)$$

Integration of each equation in (11.06) by itself quickly verifies that these components would die out to $1/e$ of their initial values after time T_2.

Upon evaluating the cross product in Eq. (11.02) and adding the contribution from Eqs. (11.05) and (11.06), one obtains the Bloch equations:

$$\begin{aligned} dM_x/dt &= \gamma[M_yH_0 + M_zH_1 \sin\omega t] - M_x/T_2 \\ dM_y/dt &= \gamma[M_zH_1 \cos\omega t - M_xH_0] - M_y/T_2 \\ dM_z/dt &= \gamma[-M_xH_1 \sin\omega t - M_yH_1 \cos\omega t] \\ &\quad + (M_0 - M_z)/T_1. \end{aligned} \quad (11.07)$$

The resonance absorption phenomenon we treat here is described largely by a particular solution of Eqs. (11.07) which is obtained in the Appendix. We shall be interested in M_x and M_z since the experimentally applied fields $2\mathbf{H}_1 \cos\omega t$ and $\mathbf{H}_0$ are in these directions. We find

$$M_x = \tfrac{1}{2}\chi_0\omega_0 T_2 \times \frac{(2H_1 \cos\omega t)T_2(\omega_0-\omega)+2H_1 \sin\omega t}{1+T_2^2(\omega_0-\omega)^2+\gamma^2H_1^2T_1T_2} \quad (11.08)$$

$$M_z = \chi_0 H_0 \frac{1+T_2^2(\omega_0-\omega)^2}{1+T_2^2(\omega_0-\omega)^2+\gamma^2H_1^2T_1T_2}, \quad (11.09)$$

where χ_0 is the Curie susceptibility in Eq. (4.07).

Comparison of Eq. (11.08) with Eq. (10.03) identifies the Bloch susceptibilities as

$$\chi' = \tfrac{1}{2}\chi_0\omega_0 T_2 \frac{T_2(\omega_0-\omega)}{1+T_2^2(\omega_0-\omega)^2+\gamma^2H_1^2T_1T_2}$$

$$\chi'' = \tfrac{1}{2}\chi_0\omega_0 T_2 \frac{1}{1+T_2^2(\omega_0-\omega)^2+\gamma^2H_1^2T_1T_2}.$$

Finally, the absorption is obtained from Eq. (10.04) and the above value of χ''.

$$A = \omega H_1^2\chi_0 \frac{\omega_0 T_2}{1+T_2^2(\omega_0-\omega)^2+\gamma^2H_1^2T_1T_2}. \quad (11.10)$$

The key equation (11.02) into which Bloch introduced the interactions involving T_1 and T_2 is a classical one, and it may well be asked why we apply it with such confidence to the small nuclear domain under consideration. In the quantum-mechanical formalism, the magnetic moment $\boldsymbol{\mu}$ is replaced by an *operator*, and a procedure is developed for finding directly from the operator and the nuclear wave function the *expectation value* of the quantity the operator represents. It is a happy fact that the quantum-mechanical expression for the time derivative of the magnetic moment operator is identical in form to Eq. (11.02), $\boldsymbol{\mu}$ being replaced by the operator. Upon proceeding to take the expectation value of each member, one alters in no way the form of the equation, which is now satisfied by the expectation value of $\boldsymbol{\mu}$. This expectation value corresponds to the observable quantity in an experiment, and therefore Eq. (11.02) describes the system no matter how large or small its quantum numbers may be.

12. The Bloch Susceptibilities

a. No saturation effects.—Consider first the situation when

$$\gamma^2H_1^2T_1T_2 \ll 1. \quad (12.01)$$

For example, this condition, when applied to the proton resonance in water at 7000 gauss, requires that H_1 be about one milligauss or smaller. The susceptibilities of Eq. (11.09) then reduce to

$$\chi' = \tfrac{1}{2}\chi_0\omega_0 T_2 \frac{T_2(\omega_0-\omega)}{1+T_2^2(\omega_0-\omega)^2} \quad (12.02)$$

$$\chi'' = \tfrac{1}{2}\chi_0\omega_0 T_2 \frac{1}{1+T_2^2(\omega_0-\omega)^2} \quad (12.03)$$

and the absorption is

$$A = 2H_1^2\omega\chi''. \quad (12.04)$$

The factor ω in Eq. (12.04) may usually be replaced by ω_0 since the resonance is ordinarily quite narrow compared to the resonant frequency ω_0.

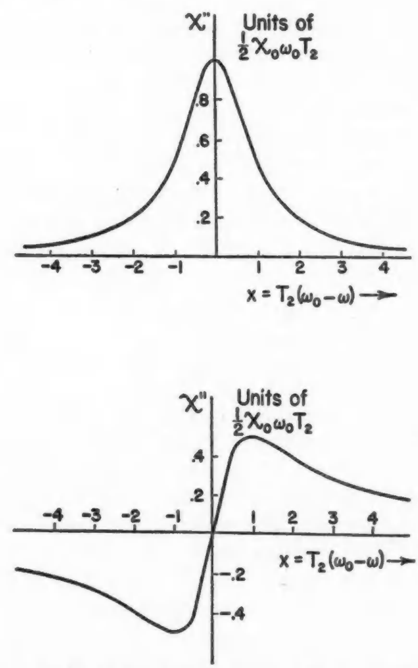

FIG. 4. The Bloch nuclear susceptibilities plotted against the dimensionless variable $x = T_2(\omega_0-\omega)$. These graphs apply to conditions of negligible saturation.

Equations (12.02) and (12.03) are plotted in Fig. 4 against an abscissa $x=T_2(\omega_0-\omega)$. The resonant character of the absorption is apparent, maximum absorption occurring at the Larmor frequency. Further, the half-width at half-maximum intensity occurs when

$$T_2|\omega_0-\omega|_{\frac{1}{2}}=1, \qquad (12.05)$$

and thus $1/T_2$ is the half-width expressed as an angular frequency. The qualitative considerations of Sec. 9 are borne out: spin-spin processes fix the line width in a perfectly homogeneous external field.

The Bloch susceptibilities have the so-called Lorentz shape[21] which was first obtained in the simple classical analysis of radiation by a damped oscillator.[22]

b. Saturation effects.—We now remove the restriction Eq. (12.01) and increase H_1 until $\gamma^2H_1^2T_1T_2$ has a value of the order of unity. Under this condition, the nuclear spin system is said to be *saturated*, that is, it is soaking up radiofrequency energy to such an extent that the relaxation processes are unable to keep it at the lattice temperature. One can observe this directly from the expression (11.09) for M_z if χ_0 is replaced by its value in Eq. (4.07). The maximum value of M_z is then

$$M_z(\omega=\omega_0)=\frac{Ng^2\mu_0^2I(I+1)H_0}{3kT(1+\gamma^2H_1^2T_1T_2)}. \qquad (12.06)$$

Equation (12.06) has a form that suggests the definition of an effective spin temperature,

$$T_S^{\mathrm{eff}}=T(1+\gamma^2H_1^2T_1T_2), \qquad (12.07)$$

since, if the constant field H_0 alone were applied at a real temperature equal to T_S^{eff}, Eq. (12.06) would give the equilibrium value of M_z which would arise. Thus the excess population in the lower energy states is reduced when $\gamma^2H_1^2T_1T_2$ is no longer negligible compared to unity.

The absorption is also affected by saturation. Whereas the expression (12.04) for A increases directly with H_1^2, the general expression (11.10) approaches the constant value

$$A(\mathrm{sat})=(\omega\omega_0\chi_0)/(\gamma^2T_1)$$

as H_1^2 is increased indefinitely.

[21] G. E. Pake and E. M. Purcell, *Physical Rev.* **74**, 1184 (1948).

[22] See, for example, R. Becker, *Theorie der Electrizität* (Edwards Brothers, Inc., Ann Arbor, 1945), p. 139.

Appendix

To aid in obtaining the particular solution of the Bloch equations, define the complex numbers

$$\mathfrak{M}_+=M_x+iM_y$$
$$\mathfrak{M}_-=M_x-iM_y.$$

First add i times the second equation of (11.07) to the first, then subtract i times the second from the first, obtaining

$$d\mathfrak{M}_+/dt=\gamma[-iH_0\mathfrak{M}_+ +iH_1\mathfrak{M}_ze^{-i\omega t}]-\mathfrak{M}_+/T_2 \qquad (1)$$

$$d\mathfrak{M}_-/dt=\gamma[iH_0\mathfrak{M}_- -iH_1\mathfrak{M}_ze^{i\omega t}]-\mathfrak{M}_-/T_2. \qquad (2)$$

We remove the time dependence by letting

$$\left.\begin{array}{l}\mathfrak{M}_+=e^{-i\omega t}\mathfrak{N}_+\\ \mathfrak{M}_-=e^{i\omega t}\mathfrak{N}_-\end{array}\right\}, \qquad (3)$$

so that Eqs. (1) and (2) become, after substituting $\omega_0=\gamma H_0$,

$$\left.\begin{array}{l}i\omega\mathfrak{N}_+=i\omega_0\mathfrak{N}_+-i\omega_0(H_1/H_0)\mathfrak{M}_z+\mathfrak{N}_+/T_2\\ i\omega\mathfrak{N}_-=i\omega_0\mathfrak{N}_--i\omega_0(H_1/H_0)\mathfrak{M}_z-\mathfrak{N}_-/T_2\end{array}\right\}. \qquad (4)$$

Equations (4) yield $\mathfrak{N}_+$ and $\mathfrak{N}_-$:

$$\left.\begin{array}{l}\mathfrak{N}_+=\dfrac{\omega_0(H_1/H_0)\mathfrak{M}_z}{\omega_0-\omega-i/T_2}\\ \mathfrak{N}_-=\dfrac{\omega_0(H_1/H_0)\mathfrak{M}_z}{\omega_0-\omega+i/T_2}\end{array}\right\}. \qquad (5)$$

To evaluate $\mathfrak{M}_z$ we require the third equation of (11.07). We seek a solution characterized by a steady-state absorption, and it will be assumed that $\mathfrak{M}_z$ is time independent. This assumption is not so restrictive as to require that the spin and lattice temperatures be equal, but merely requires that the spin system reach equilibrium determined by the inflow of r-f energy, the relaxation times, and the lattice temperature; this equilibrium state may well correspond to a spin temperature appreciably higher than the lattice temperature.

The assumption $d\mathfrak{M}_z/dt=0$ combines with the identity

$$M_x\sin\omega t+M_y\cos\omega t=(1/2i)(\mathfrak{M}_+e^{i\omega t}-\mathfrak{M}_-e^{-i\omega t})$$

to yield from Eq. (12.07) that

$$\frac{\gamma H_1}{2i}[\mathfrak{M}_+ e^{i\omega t} - \mathfrak{M}_- e^{-i\omega t}] = \frac{\mathfrak{M}_z - M_0}{T_1}, \quad (6)$$

which on further rearranging becomes

$$\frac{M_0 - \mathfrak{M}_z}{T_1} = \omega_0(H_1/H_0)\left(\frac{\mathfrak{N}_+ - \mathfrak{N}_-}{2i}\right). \quad (7)$$

The bracket of Eq. (7) is found directly from Eq. (5); it is real, allowing M_z to replace $\mathfrak{M}_z$ and, upon solving the resultant expression for M_z, one obtains

$$M_z = \chi_0 H_0 \frac{(\omega_0 - \omega)^2 + 1/T_2^2}{(\omega_0 - \omega)^2 + (1/T_2^2) + (\gamma^2 H_1^2 T_1/T_2)}. \quad (8)$$

The quantity M_x may be obtained from

$$M_x = \tfrac{1}{2}(\mathfrak{M}_+ + \mathfrak{M}_-) = \tfrac{1}{2}[\mathfrak{N}_+ e^{-i\omega t} + \mathfrak{N}_- e^{i\omega t}]$$

and Eqs. (5) as

$$M_x = \tfrac{1}{2}\omega_0(H_1/H_0)M_z\left[\frac{e^{-i\omega t}}{\omega_0 - \omega - i/T_2} + \frac{e^{i\omega t}}{\omega_0 - \omega + i/T_2}\right]. \quad (9)$$

Since M_z is real and the terms in brackets are complex conjugates, the expression (9) is indeed real, and

$$M_x = \tfrac{1}{2}\omega_0 \chi_0 T_2 \times \left[\frac{(2H_1 \cos\omega t)T_2(\omega_0 - \omega) + 2H_1 \sin\omega t}{1 + T_2^2(\omega_0 - \omega)^2 + \gamma^2 H_1^2 T_1 T_2}\right]. \quad (10)$$

List of Principal Symbols

$\boldsymbol{\mu}$ = magnetic moment vector
$\mathbf{p}$ = angular momentum vector
c = velocity of light in vacuum
M = mass of proton
e = charge on the proton
g = nuclear g-factor (dimensionless)
$\hbar = (2\pi)^{-1}$ times the Planck constant
$\mathbf{H}$ = magnetic field
$\mathbf{I}$ = nuclear spin vector (dimensionless)
m = magnetic quantum number (dimensionless)
I = nuclear spin (maximum value of m)
$\mathbf{L}$ = torque vector
ω_0 = angular frequency of Larmor precession
$U(m)$ = Zeeman energy of a magnetic moment in state m
$\mu_0 = e\hbar/(2Mc)$ = nuclear magneton
$\mu = gI$ = dimensionless number of nuclear magnetons: "the nuclear magnetic moment"
$\mathbf{B}$ = magnetic induction vector
$\mathbf{M}$ = volume density of magnetization
χ = magnetic susceptibility (dimensionless)
N = number of nuclear magnetic moments per unit volume
k = Boltzmann constant
T = absolute temperature, usually of the lattice
$N(m)$ = number of nuclei per unit volume in state m
χ_0 = nuclear Curie susceptibility
ω = angular frequency of incident radiation
$\mathbf{H}_1$ = rotating magnetic field
$\mathbf{H}_0$ = constant magnetic field vector
$P(m \rightarrow m')$ = transition probability per unit time
ν = frequency of radiation
$\rho(\nu)$ = energy density of radiation at frequency ν
$C(m \rightarrow m')$ = absolute transition probability
T_1 = spin-lattice relaxation time
T_S = temperature of the nuclear spin system
$W(p \rightarrow q)$ = total transition probability per unit time after detailed balancing
n = excess of nuclei per unit volume in lower Zeeman state (for $I = \frac{1}{2}$)
n_0 = equilibrium value of n for $T_S = T$
H_{loc} = magnetic field produced at a lattice point by neighboring nuclear moments
A = energy absorbed per second per unit volume
χ', χ'' = real and imaginary parts of $\chi = \chi' - i\chi''$
M_0 = equilibrium value of z-component of nuclear magnetization
T_2 = time for a freely precessing component of nuclear magnetization to fall to e^{-1} of its value; inverse measure of line width
$\gamma = ge/(2Mc)$ = ratio of angular Larmor frequency to magnetic field
$x = T_2(\omega_0 - \omega)$ = dimensionless variable
$\mathcal{L}$ = complex inductance of coil containing nuclear sample
$\mathcal{Y}$ = complex admittance
$\mathcal{V}$ = complex potential difference
a = ratio of modulation amplitude (gauss) to line width (gauss)
G_r = gain of the receiver
D = deflection of "lock-in" meter
$s = \gamma^2 H_1^2 T_1 T_2$ = saturation factor
β = angular frequency of magnetic field modulation
ΔH = difference between H and its resonant value for applied radiofrequency ν
$p(\Delta H)$ = line shape for interacting pair
$f(\nu)$ = any normalized line shape on a frequency scale
$\langle(\Delta\nu)^2\rangle_{\mathrm{Av}}$ = second moment of line shape
τ_c = correlation time
δH = separation (in gauss) between points of extreme slope of the absorption curve.

The Allowed Beta-Spectrum

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I. Introduction

The development of the theory of beta-decay and of the experimental work done in connection with this theory forms an interesting example of the gradual evolution of new ideas, both experimental and theoretical, which led and still leads to a compact and useful theory of the physical process involved. In this case, as in others, progress was slow; however, each contribution whether in support of or in disagreement with this theory has formed a resulting foundation for beta-decay which at this time seems solid.

It is the purpose of the following discussion to trace the experimental and theoretical investigations of the allowed beta-decay process. In addition to allowed transitions there are also forbidden transitions just as in the case of atomic spectra. The forbidden transitions will not be treated at any length in the present work, since their peculiarities present a problem in themselves.

II. The Fundamental Nature of the Continuous Spectrum

Soon after the discovery of radioactivity the radiations from radioactive nuclei were found to consist of three distinct types.[1] These three types were called alpha-, beta-, and gamma-radiations in accordance with their ability to penetrate matter. Subsequent investigations revealed that the alpha-particle consists of a helium nucleus, that the beta-particle is identical with an electron, and that the gamma-ray is electromagnetic radiation.

Early studies of the spectra of the alpha-radiations showed that these particles were emitted in monoenergetic groups. At the same time it was observed that many of the electron spectra which accompanied alpha-disintegration presented a series of monoenergetic lines. For this reason the assumption was made that beta-radiations were also emitted in monoenergetic groups.

The first serious discrepancies resulting from the assumption that beta-rays were emitted in discrete groups, as opposed to a continuous distribution, arose when Hahn and Meitner[2] found that no monoenergetic lines appeared in the beta-spectrum of Radium E. Instead, the spectrum on their photographic plate showed a continuous background which seemed to extend from a lower momentum[3] limit of 1600 gauss-cm to an upper momentum limit of approximately 5000 gauss-cm. This continuous background on the photographic plate had a maximum intensity in the region of 2100 gauss-cm.

It is now known that this continuous spectrum is the distribution which occurs in the beta-decay process. The monoenergetic lines, which had been regarded as illustrative of the process of beta-decay, are in reality the radiations which result when an atomic electron is ejected in lieu of a nuclear gamma-ray. This electron radiation is customarily referred to as arising from internal conversion. Figure 1 illustrates a continuous beta-distribution accompanied by superimposed internal conversion lines.

Following the work of Hahn and Meitner[2] many possible explanations were proposed in order to account for the existence of a continuous distribution.[4] According to most of these pro-

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[1] For a summary of the experiments leading to the discovery of radioactivity and of the work done in this field for the first few years following its discovery, see G. E. M. Jauncey, *Am. J. Physics* **14**, 226 (1946).

[2] O. Hahn and L. Meitner, *Physikalische Zeits.* **9**, 321, 697 (1908).

[3] It is customary in beta-ray spectroscopy to express momentum $p=mv$ as the product of the homogeneous magnetic induction B and the radius of curvature ρ of the orbit of the beta-particle in this field. From the Lorentz equations of motion one then obtains

$$evB/c=mv^2/\rho,$$

giving

$$mv=p=e\{B\rho\}/c.$$

It should be noted that the momentum units gauss-cm differ from the conventional c.g.s. units of momentum by the factor e/c. Habitual usage has caused momentum to be referred to as $H\rho$ rather than $B\rho$.

[4] Moseley, *Proc. Roy. Soc.* **A87**, 230 (1912); Meitner, *Zeits. f. Physik* **19**, 307 (1923); see also Ellis and Wooster, *Proc. Camb. Phil. Soc.* **22**, 849 (1925) for a discussion of possible effects.

posals the continuous distribution of electron energies occurred as a consequence of the degradation in energy of the nuclear beta-particle. Under these circumstances the energy of the beta-particle would be shared by one or more of the atomic electrons, or by electromagnetic radiation. The total energy of these radiations should then equal the energy available to the supposed monoenergetic nuclear radiations.

However, further experimental work by Emeléus,[5] showed that the number of electrons in the Radium E spectrum was equal to the number of alpha-particles from Radium F when Radium E and Radium F were in equilibrium. This established the fact that the number of beta-particles which appear in the continuous distribution equals the number of disintegrations, in conflict with the above-mentioned proposals.

These results of Emeléus led Ellis and Wooster[6] to study by means of a calorimeter the average energy associated with the disintegration of Radium E. According to measurements of the beta-distribution of Radium E at that period, the distribution had an approximate upper energy limit of 1.05 Mev and had an average energy of 0.39 Mev. Ellis and Wooster pointed out that if the spectrum was actually discrete the average energy as measured calorimetrically should be approximately 1.0 Mev; however, if the spectrum was truly continuous as observed, the average energy would be 0.39 Mev.

The calorimetric measurements by Ellis and

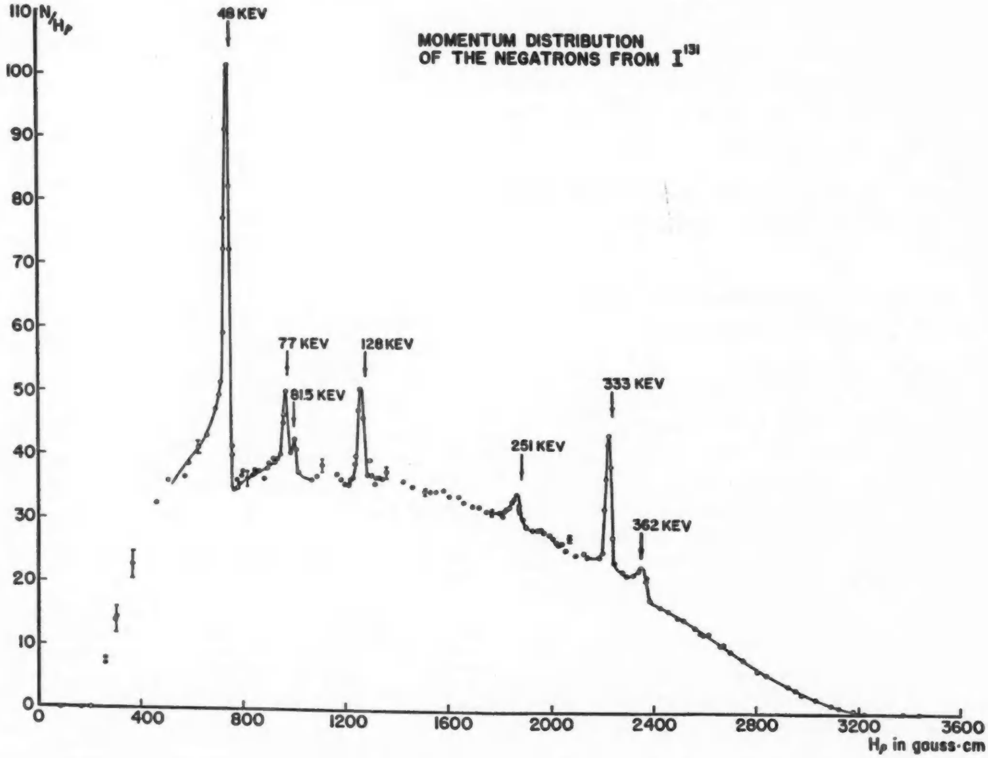

FIG. 1. The continuous beta-spectrum and internal conversion electron spectrum of I^{131}. This illustrates the coexistence of continuous and discrete electron spectra.

[5] K. G. Emeléus, *Proc. Camb. Phil. Soc.* **22**, 400 (1924).
[6] C. D. Ellis and W. A. Wooster, *Proc. Roy. Soc.* **A117**, 109 (1928). See also L. Meitner and W. Orthmann, *Zeits. f. Physik* **60**, 143 (1930).

Wooster[6] indicated that the average energy per beta-disintegration of Radium E was 0.35 Mev in excellent agreement with the average energy as calculated from the observed continuous beta-spectrum.

This evidence implies that either energy is not conserved in the beta-decay process or some of the energy released in this process escapes from the immediate surroundings of the radioactive nucleus in the form of radiation which could not be detected by the methods used at that time.

III. The Neutrino Hypothesis and the Fermi Theory of Beta-Decay

As a result of the calorimetric experiments concerning the average energy of the beta-particles, Pauli suggested that energy is conserved in the beta-decay process and that the undetected energy is carried away by a neutral particle of small mass, which is now called the ***neutrino***.

Using this suggestion Ellis and Mott[7] postulated that the energy difference between the parent and daughter nuclei in beta-decay is equal to the maximum possible kinetic energy of the emitted beta-particle plus the energy equivalent of its rest mass and that of the neutrino. This assumption was later to be found correct and now occupies an important position in our ideas concerning nuclear structure.

In 1934 on the basis of the neutrino hypothesis, Fermi proposed his well-known theory of beta-decay. According to the Fermi theory the assumption is made that a neutrino and a beta-particle are emitted simultaneously in the disintegration of the radioactive nucleus. The kinetic energy of the beta-particle plus the kinetic energy of the associated neutrino is equal to the maximum kinetic energy of the beta distribution; therefore, the concept of discrete energy levels within the nucleus is maintained. The maximum energy of the beta-distribution is then a measure of the total energy available for a given transition. Because of the large mass of the nucleus compared to that of the electron or neutrino, the requirements for both conservation of momentum and conservation of energy allow the recoil nucleus to assume only a negligible fraction of the total kinetic energy available to the system.[8] Therefore, the kinetic energy of the recoil nucleus as well as the energy of the soft x-ray photons which are emitted because of the electron-nucleus charge acceleration is usually ignored in the energy balance.

In this process the parent nucleus is assumed to decay to the daughter by a change in its total charge Z of ± 1. If the charge increases by unity, an electron and an antineutrino are emitted.[9] When the nuclear charge decreases by unity a positron and a neutrino are emitted.

The transition probability per energy interval per unit time for electron-antineutrino emission from a beta-active nucleus (or the number of particles emitted per energy interval per unit time) can now be calculated by the time dependent perturbation procedure of quantum mechanics.[10] This calculation results in an expression which consists of two main factors: one, a statistical factor $S(W)$ dependent upon the total (kinetic plus rest) electron energy W; the other a matrix element[10] (ME) containing the wave functions of the parent nucleus, the daughter nucleus, the negatron (or positron), and the antineutrino (or neutrino) along with the proper interaction terms between the nucleons and leptons.[11]

The transition probability can be written as the probability per unit time for the emission of a beta-particle with energies between W and $W+dW$. This becomes

$$P(Z, W)dW = (2\pi/h)\,|(ME)|^2 S(W)dW. \quad (1)$$

The function $S(W)$ is the density of the total number of energy states available and is the product of the density of electron energy states (ρ_e) and the density of neutrino energy states (ρ_ν). Mathematically formulated this becomes

$$S(W)dW = \rho_e \rho_\nu dW. \quad (2)$$

[7] C. D. Ellis and N. F. Mott, *Proc. Roy. Soc.* **A141**, 502 (1933).

[8] For a detailed discussion of the experiments concerning the detection of the recoil nuclei see J. S. Allen, *Am. J. Physics* **16**, 451 (1948) and H. R. Crane, *Rev. Mod. Physics* **20**, 278 (1948).

[9] For the purpose of the present discussion the properties of the neutrino and antineutrino can be considered as identical.

[10] For a discussion of this method see Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., 1949), pp. 189 ff.

[11] Lepton refers to that class of small particle to which the negatron, positron, neutrino and antineutrino belong.

In order to develop $S(W)$ mathematically it is convenient to introduce the following terms:

W = the total (rest plus kinetic) electron energy in m_0c^2 units, where m_0 is the rest mass of the electron,
p = the momentum of the electron in m_0c units,
W_η = the total neutrino energy in m_0c^2 units,
η = the momentum of the neutrino in m_0c units, and

$$W_0 = W + W_\eta = \text{maximum electron energy.} \quad (3)$$

In these units the relativistic relations between total energy and momentum are

$$W^2 = p^2 + 1, \quad (4)$$

and

$$W_\eta^2 = \eta^2 + (\mu/m_0)^2,$$

where μ = the rest mass of the neutrino. Since μ is assumed to be zero,

$$W_\eta^2 = \eta^2. \quad (5)$$

The quantity $\rho_e dW$ can now be derived from these relations. The total number of electron states in a given volume of phase space is equal to the ratio of that volume to the smallest volume h^3 in which an electron can exist. Therefore, if we consider a volume element in a momentum space $(4\pi p^2 dp)$, the total number of states in this volume (between W and $W+dW$) is

$$\rho_e dW = 4\pi p^2 dp/h^3. \quad (6)$$

From Eq. (4)

$$pdp = WdW.$$

Therefore, Eq. (6) can be written

$$\rho_e dW = (4\pi/h^3) pWdW. \quad (7)$$

In a similar manner, the total number of neutrinos between η and $\eta + d\eta$ becomes

$$\rho_\eta dW_\eta = (4\pi/h^3)\eta^2 d\eta,$$

and then the density of neutrino states is

$$\rho_\eta = (4\pi/h^3)\eta^2 d\eta/dW_\eta, \quad (8)$$

and since $\eta^2 = W_\eta^2$ for $\mu \rightarrow 0$,

$$\rho_\eta = (4\pi/h^3) W_\eta^2 = 4\pi(W_0 - W)^2/h^3. \quad (9)$$

Therefore, inserting Eqs. (9) and (7) into Eq. (2):

$$S(W)dW = (16\pi^2/h^6) pW(W_0 - W)^2 dW = (16\pi^2/h^6)(W^2-1)^{\frac{1}{2}} W(W_0 - W)^2 dW. \quad (10)$$

Equation (10) presents the statistical factor. It now is necessary to consider the matrix element (ME). Since (ME) involves an integration over the nuclear volume and since the wavelengths of the electron and neutrino (or antineutrino) are large compared to the dimensions of the nuclear volume, the electron and neutrino wave functions can be considered as uniform over the nuclear volume. If, with this approximation, the element (ME) does not vanish, the transition is said to be allowed; in general, the element (ME) so obtained does not vanish if the difference between the intrinsic spin and angular momentum of the parent and daughter nuclei is zero or one (for the so-called "Gamow-Teller" selection rules—tensor or axial vector interaction). The Fermi function $F(Z, p)$ is derived from the absolute square of the electron and neutrino wave functions.[12]

$$F(Z, p) = \frac{4(2pR)^{2S-2} e^{\pi\alpha ZW/p} |\Gamma(S + i\alpha ZW/p)|^2}{|\Gamma(2S+1)|^2}, \quad (11)$$

where

R = the nuclear radius,
Z = the charge of the *daughter* nucleus,
α = the atomic fine structure constant (1/137), and
$S = (1 - \alpha^2 Z^2)^{\frac{1}{2}}$.

The Fermi function $F(Z, p)$ possesses this form as a result of the distorting effect of the Coulomb potential on the otherwise plane wave character of the wave function of the electron.[13]

In the case of elements of low atomic number (normally considered as $Z \leq 30$) the quantity S approaches unity and as a result $F(Z, p)$ can be approximated[14] as

$$F(Z, p) \cong \frac{2\pi Z\alpha W/p}{1 - e^{-2\pi Z\alpha W/p}}. \quad (12)$$

That part of the matrix element, in an allowed transition, which remains inside the integral over

[12] E. J. Konopinski, *Rev. Mod. Physics* **15**, 209 (1943).

[13] For the solution of the nonrelativistic equation of motion see Jeffries, *Mathematical physics* (Oxford University Press, London, 1946), pp. 586–587. For the relativistic wave functions in the continuous spectrum for the Coulomb field see M. E. Rose, *Physical Rev.* **51**, 484 (1937).

[14] Kurie, Richardson, and Paxton, *Physical Rev.* **48**, 167 (1935); *Physical Rev.* **49**, 368 (1936).

the nuclear volume contains only the initial and final nuclear wave functions and the proper interaction term (tensor or axial vector). This integral is independent of the energy of the emitted electron and neutrino.

Thus, for allowed transitions the Fermi theory of beta-decay predicts that the number of beta-particles emitted per unit time with energies between W and $W+dW$ is

$$P(Z,W)dW=(\text{const.})F(Z,p)\times(W^2-1)^{\frac{1}{2}}W(W_0-W)^2dW. \tag{13}$$

The differentiation between allowed and forbidden spectra can be obtained in a more general manner by the use of the so-called *ft* value. This term *ft* can be defined as follows: The total probability of beta-decay per unit time is proportional to the inverse of the half-life. If $t=$ the half-life, then from Eq. (13)

$$\frac{\log 2}{t}=\int_1^{W_0}P(Z,W)dW=(\text{const.})f(Z,W_0), \tag{14}$$

and

$$f(Z,W_0)t=\text{a constant}. \tag{15}$$

This product of $f(Z,W_0)t$ has a numerical value which falls within general limits depending on whether the transition is allowed or forbidden.[15]

IV. The Konopinski-Uhlenbeck Modification

Experimental evidence[16] obtained about the time that Fermi proposed his theory appeared to indicate that the measured beta-distributions did not agree with the Fermi theory. Konopinski and Uhlenbeck, in a paper[17] discussing the Fermi theory, pointed out that the interaction Hamiltonian between nucleons and leptons which Fermi used was not unique. Consequently, they proposed a modification of this interaction Hamiltonian wherein the neutrino wave function was replaced by its first derivative.

The effect of this modification on the transition probability was to multiply the Fermi distribution by a factor $(W_0-W)^2$, giving

$$P(Z,W)dW=(\text{const.})F(Z,p)\times(W^2-1)^{\frac{1}{2}}W(W_0-W)^4dW.$$

This distribution was shown to agree well with the experimental evidence available at this period. Further measurements, particularly those of Kurie, Richardson, and Paxton,[14] suggested satisfactory agreement with the shape of the distribution given by the K-U modification and showed marked discrepancies from the distribution predicted by the Fermi theory.

Although it was later shown that source thickness and straggling of the beta-particles in the gas of the cloud chamber greatly distorted the spectrum, the work of Kurie, Richardson, and Paxton[14] made an important contribution to the study of beta-spectra. This group introduced a new method of analysis of the shapes of the continuous beta-distributions. This method is now well known as the F-K (Fermi-Kurie) plot.

Since the number $N(p)$ of particles emitted per unit time per unit momentum interval is proportional to the probability function $P(p)$, the Fermi distribution can be plotted in the following manner: Consider

$$N(p)dp=(\text{const.})p^2F(Z,p)(W_0-W)^2dp;$$

then

$$N(p)/\{p^2F(Z,p)\}^{\frac{1}{2}}=(\text{const.})(W_0-W).$$

Thus, when $N(p)/\{p^2F(Z,p)\}^{\frac{1}{2}}$ is plotted against energy a straight line should result if the distribution obeys the Fermi theory. On the other hand, if $N(p)/\{p^2F(Z,p)\}^{\frac{1}{4}}$ is plotted against energy, agreement of the distribution with the K-U modification would yield a straight line.

By use of this analysis, Kurie, Richardson, and Paxton[14] showed that the experimental results obtained from their cloud chamber histograms gave a linear plot only for the K-U modification. For the Fermi theory, the plot was convex toward the energy axis. Other experimental results[16] also seemed to favor the K-U modification of the Fermi theory of beta-decay and consequently for a time the K-U modification seemed fairly well established.

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[15] For a tabulation of the *ft* values of a number of radioactive beta-transitions and for their empirical classification into allowed and forbidden types see reference 12.

[16] Alichanow, Alichanian, and Dzelepow, *Zeits. f. Physik* **93**, 350 (1935). Ellis and Henderson, *Proc. Roy. Soc.* **A146**, 206 (1934).

[17] E. J. Konopinski and G. E. Uhlenbeck, *Physical Rev.* **48**, 7 (1935).

V. Restoration of the Fermi Theory

In 1939 Tyler[18] studied the positron and negatron spectra of Cu^{64} using sources and source backings of various thicknesses. He found that the distortions introduced by very thick sources (23 mg/cm^2) produced continuous spectra the shapes of which resembled those predicted by the K-U modification. However, thinner sources of Cu^{64} (4.7 and 2.2 mg/cm^2) gave spectral shapes that favored the Fermi theory in the high energy region. These curves of Tyler's are shown in Fig. 2. The conclusions drawn from these experiments were that thin sources gave data in the high energy region favoring the Fermi theory, but the spectra seemed to deviate from the Fermi theory to a much greater extent in the low energy region than could be accounted for by the source thickness.

At the same time Lawson[19] pointed out in a study of P^{32} and Na^{24} that only Kurie plots based on the Fermi theory gave extrapolated maximum energies which were in agreement with the end points found in the corresponding continuous momentum distribution. In the case of both P^{32} and Na^{24} Lawson found that a Kurie extrapolation based on the K-U modification gave end points which were 20 to 40 percent higher than the maximum energy observed experimentally.

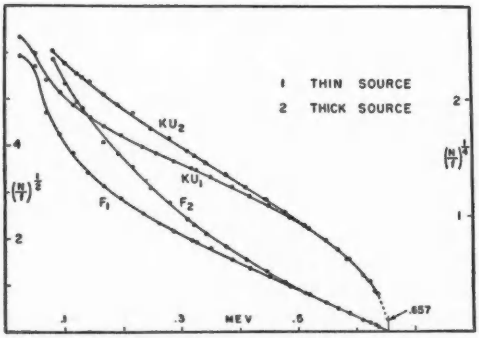

FIG. 2. The Kurie plots for the positrons from Cu^{64} according to Tyler (see reference 18). The curves F_2 and F_1 are F-K plots made in accordance with the Fermi theory using two sets of data resulting, respectively, from the use of a thick source and a somewhat thinner source. The same data treated in accordance with the K-U modification give the curves KU_2 and KU_1.

Further work by Lawson and Cork[20] and Townsend[21] supported the original Fermi formulation.

Further evidence against the K-U modification has appeared in the study of K-capture/positron ratios for transitions that can occur by either positive beta-decay or orbital electron capture.[22]

VI. The Final Tests of the Fermi Theory

Up to this point work on allowed spectra had favored agreement with the Fermi theory in the high energy region; however, the thinnest sources employed failed to give linear F-K plots, all showing an excess of particles in the low energy regions. In particular, Tyler[18] had pointed out that the deviation from the Fermi theory in the positron spectrum of Cu^{64} was much greater than the deviation in the case of the negatron spectrum. Since it was felt that an instrumental distortion should cause deviations of the same order of magnitude for both positrons and negatrons, the Fermi theory was assumed inadequate in the low energy region.

It was not until 1945 that further tests of the Fermi theory were performed. Employing an electrostatic spectrometer Backus[23] measured the ratio of positrons to negatrons in the disintegration of Cu^{64}. The basis of this study rested on the assumption that an instrumental distortion should not alter the true ratio of positrons to

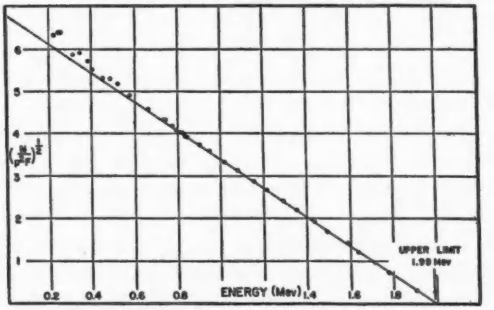

FIG. 3. The F-K plot of Lawson and Cork's (see reference 20) data on In^{114}. The linearity over so large a portion of the spectrum strongly supports the original Fermi theory.

[18] A. W. Tyler, *Physical Rev.* **56**, 125 (1939).
[19] J. L. Lawson, *Physical Rev.* **56**, 131 (1939).
[20] Lawson and Cork, *Physical Rev.* **57**, 982 (1940).
[21] A. A. Townsend, *Proc. Roy. Soc.* **A177**, 357 (1941).
[22] Bradt, Gugelot, Huber, Medicus, Preiswerk, and Scherrer; *Physical Rev.* **68**, 57 (1945); *Helv. Physica Acta* **18**, 351 (1945).
[23] J. Backus, *Physical Rev.* **68**, 59 (1945).

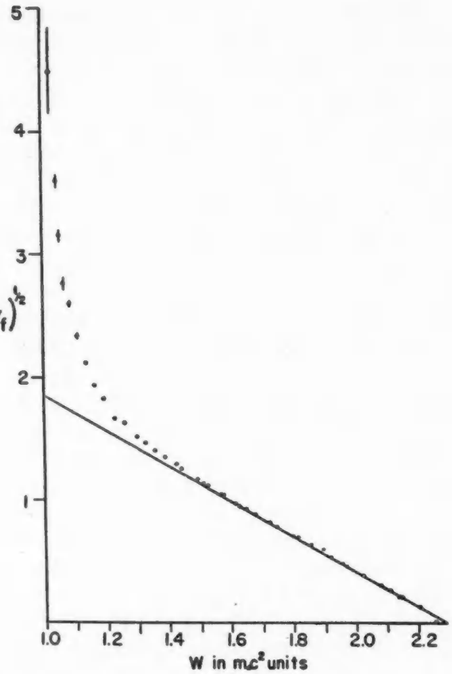

FIG. 4. The F-K plot of the Cu^{64} positron spectrum as obtained by Cook and Langer (see reference 26). Because of the small *average* source thickness, the deviation from linearity was for a time thought to be a real effect.

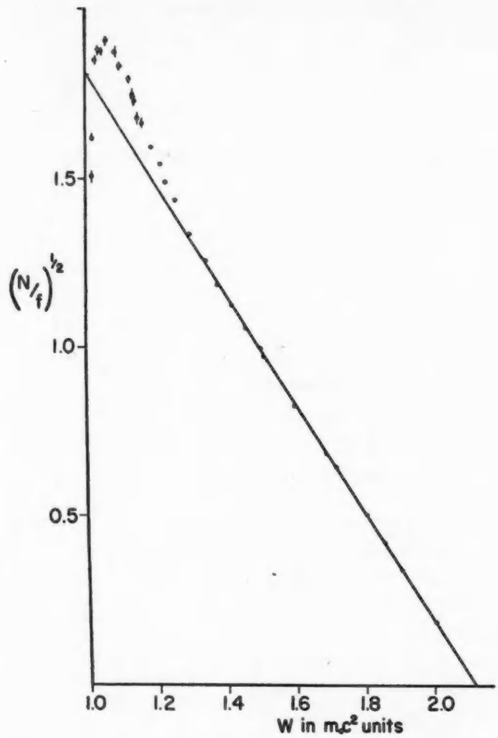

FIG. 5. The F-K plot of the Cu^{64} negatron spectrum according to Cook and Langer (see reference 26). The same sources were used as in Fig. 4.

negatrons, whereas a failure of the Fermi theory would quite possibly result in a different ratio. The study was restricted to the low energy region from 15 kev to 50 kev; the lower limit was imposed by the window of the Geiger-Mueller counter and the upper limit by the maximum potential of the electrostatic spectrometer. The sources which Backus used were thin when compared to those used by Tyler and were mounted upon a 0.03-micron collodion backing. The results of this experiment showed a ratio of positrons to negatrons from Cu^{64} which was quite a bit larger than could be expected from the Fermi theory. Because of the care taken in the preparation of the source it appeared at the time that this discrepancy might be real.

Cranberg and Halpern[24] pointed out that Backus' primary assumption, that scattering distortions should be of an equivalent order of magnitude for positrons or negatrons, was probably not correct. This group suggested that because the shapes of the positron and negatron distributions of Cu^{64} were very different in the low energy regions, the effects of scattering would be different in such a way as to accentuate the excess of positrons.

It was evident that additional data, using thin sources and thin backings, concerning the complete distributions of Cu^{64} was needed. Employing a large radius of curvature high resolution spectrometer,[25] Langer and Cook carried out such a study.[26] Sources were deposited from solution onto thin Zapon backings by evaporation of the solvent and subsequent crystalization of the source. These were then weighed to obtain their

[24] L. Cranberg and J. Halpern, *Physical Rev.* **73**, 259 (1948).

[25] L. M. Langer and C. S. Cook, *Rev. Sci. Inst.* **19**, 257 (1948).

[26] C. S. Cook and L. M. Langer, *Physical Rev.* **73**. 601 (1948).

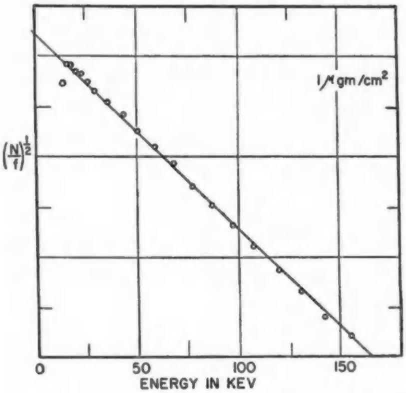

FIG. 6. The F-K plot of the negatrons from S^{35} according to Albert and Wu (see reference 31). This spectrum was the first to show the absence of a deviation in the low energy region when very thin sources are used.

average thickness. The measurements indicated average thicknesses ranging from less than 0.1 mg/cm^2 to 0.3 mg/cm^2. All such sources, although of different thicknesses, gave very nearly the same spectra, spectra which were in agreement with the Fermi theory over a large portion of the distribution but exhibited an excess of particles in the low energy regions.

To carry this type of study further the positron spectra of Cu^{61} and N^{13} were investigated under similar instrumental conditions. The largest deviation from the Fermi theory was encountered in the positron spectrum[27] of Cu^{61} which had been reported[28] to be free of nuclear gamma-radiation. This spectrum was assumed to be simple and the apparent deviation from theory appeared as an excess of positrons below 500 kev. However, the positron spectrum[29] of N^{13}, which possesses approximately the same end point as Cu^{61}, displayed no significant deviation from the Fermi theory to as low an energy as 100 kev.

The evidence did not establish either the success or failure of the Fermi theory in describing the process of beta-decay, but it did serve to stimulate a general investigation of the problem. The question of experimental technique remained; was it still possible to have instrumental distortions when such thin sources and high resolution were used?

A possible solution was offered by Owen and Primakoff.[30] Their discussion considered the distortions which could be expected in a continuous spectrum if one takes into account the shapes of internal conversion lines when sources of equivalent thickness are employed. By correcting the continuous distributions of both positrons and negatrons of Cu^{64} they showed that the deviations encountered in the continuous spectra were of the same order of magnitude as the distortions which could be anticipated to be present by analysis of the low energy tails of internal conversion lines.

Experimental technique was now re-examined. Albert and Wu,[31] utilizing sources whose average thicknesses were of the order of magnitude of one microgram per square centimeter and employing a large solenoid spectrometer of very high gathering power, were able to show that the F-K plots of S^{35} were linear over the entire range of the spectrum. However, when the spectra of the positrons and negatrons of Cu^{64} were examined (see reference 34) by this group the deviations within the low energy region were still significant.

A correction to the Fermi function to take account of the shielding of the nucleus by the atomic electrons had been derived by Rose[32] in 1936. This correction was recomputed by Longmire and Brown[33] for several elements, and was shown to be appreciable only in the extremely low energy regions.

Wu and Albert[34] applied this correction to their data on Cu^{64}. Although the low energy deviations remained appreciable in both spectra, the fact that the negatron correction increased the number of low energy negatrons and the positron correction decreased the number of low energy positrons produced positron to negatron ratios which were in closer agreement with theory.

[27] C. S. Cook and L. M. Langer, *Physical Rev.* **74**, 227 (1948).
[28] W. Gentner and E. Segrè, *Physical Rev.* **55**, 814 (1939). Bradt, Gugelot, Huber, Medicus, Preiswerk, and Scherrer, *Helv. Physica Acta* **18**, 252 (1945).
[29] Cook, Langer, Price, Jr., and Sampson, *Physical Rev.* **74**, 502 (1948).
[30] G. E. Owen and H. Primakoff, *Physical Rev.* **74**, 1406 (1948).
[31] R. D. Albert and C. S. Wu, *Physical Rev.* **74**, 847 (1948).
[32] M. E. Rose, *Physical Rev.* **49**, 727 (1936).
[33] C. Longmire and H. Brown, *Physical Rev.* **75**, 1102 (1949).
[34] C. S. Wu and R. D. Albert, *Physical Rev.* **75**, 1107 (1949).

The primary difficulty, which arises when sources are prepared by allowing the salts to crystallize from a solution, seems to be caused by nonuniform crystallization. Thus the average thickness which is obtained by weighing is not a true measure of the thickness encountered by the beta-particle.

In order to test these concepts it would indeed seem necessary to prepare very thin and extremely uniform beta-sources. A device in which the source is thermally evaporated onto its backing should produce these characteristics. The first source prepared by thermal evaporation[35] was that of Cu^{61}. The choice of Cu^{61} was made because the deviation of this spectrum from the Fermi theory occurred at a higher energy (about 500 kev) than equivalent deviations of other isotopes, and therefore the effects of a variation in source thickness should be easily detectable.

The results obtained[35] showed that the low energy excess remained in the positron spectrum of Cu^{61}. However, when Cu^{64} was studied the sources prepared by thermal evaporation, both the positron and negatron spectra[36,37] were found

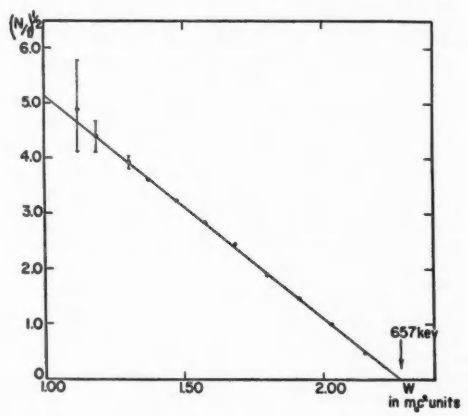

FIG. 7. The F-K plot of the positrons from Cu^{64} as obtained by the use of a very thin and uniform source (prepared by thermal evaporation). The linearity observed in this case illustrates the need for uniform sources. Even though chemical deposition of sources may produce very small average thicknesses, nonuniform crystallization will, in general, create widely varying local thicknesses.

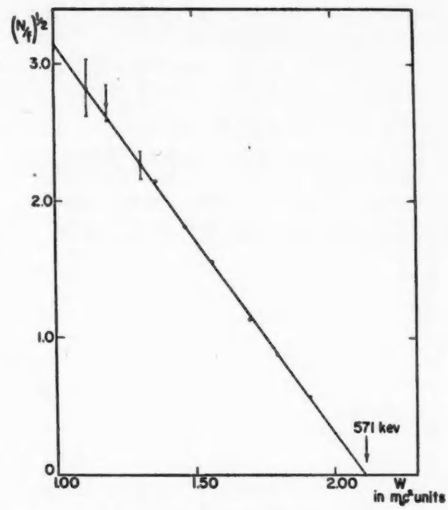

FIG. 8. The F-K plot of the negatrons from Cu^{64}. The same source was used in obtaining the above data as was employed in Fig. 7.

to agree with the Fermi theory to energies as low as 50 kev.

Autoradiographs of these evaporated sources[36] showed complete uniformity in the distribution of the radioactive material, whereas, similar investigations of chemically deposited sources[36] indicated variations in intensity of as much as 100 to 1.

Subsequent work[38] has shown the existence of nuclear gamma-rays in the disintegration of Cu^{61}. As a result the disintegration of Cu^{61} can no longer be assumed to possess a simple decay scheme. This quite probably accounts for the deviation[35] from linearity found in the F-K plot of the positrons from Cu^{61}.

An ideal source for beta-ray studies would, of course, be a gaseous source whose radiations would not have to be transmitted through any solid material. This condition was realized in the studies of tritium. By means of large proportional counters Curran, Angus, and Cockroft[39] obtained F-K plots of tritium which were linear as low as 1000 ev.

Further studies of other allowed type beta-ray

[35] G. E. Owen and C. S. Cook, *Physical Rev.* **76**, 1536 (1949).
[36] Langer, Moffat, and Price, Jr., *Physical Rev.* **76**, 1725 (1949).
[37] G. E. Owen and C. S. Cook, *Physical Rev.* **76**, 1726 (1949).
[38] Boehm, Blaser, Marmier, and Preiswerk, *Physical Rev.* **77**, 295 (1950); Owen, Cook and Owen, *Physical Rev.* **78**, 686 (1950).
[39] Curran, Angus, and Cockroft, *Phil. Mag.* **40**, 53 (1949).

transitions have indicated linearity in the F-K plots to very low energies.[40–42]

A final survey of the experimental evidence now strongly supports the Fermi theory of beta-decay for allowed type transitions. All other experimental evidence, such as the investigations of the forbidden beta-spectra and the K-capture/positron ratios, now also appear to be in good agreement with the above mentioned conclusion.

We wish to thank Dr. H. Primakoff and Dr. P. S. Jastram for their helpful comments and suggestions. This article was written as part of a program of research supported by the ONR and the AEC.

[40] Price, Motz, and Langer, *Bull. Am. Physical Soc.* **24**, No. 7, 10 (1949).
[41] Lidofsky, Macklin, and Wu, *Physical Rev.* **76**, 1888 (1949).
[42] Perez-Mendez and Brown, *Physical Rev.* **77**, 404 (1950).

State University of Iowa Colloquium of College Physicists

The eleventh annual Colloquium of College Physicists and the Associated June Lectures were held at the State University of Iowa on June 14 to 17, 1950. PROFESSOR G. W. STEWART, *State University of Iowa*, was in charge of arrangements for the meeting. All meetings were held in the Physics Building; housing was provided for the visitors by the University in Hillcrest Dormitories and Currier Hall.

The Colloquium Dinner was held in the River Room of the Iowa Union on Thursday evening, June 15th. DR. BERNARD B. WATSON, Specialist for Physics, Division of Higher Education, *United States Office of Education*, addressed the guests upon the topic "Current Activities of the United States Office of Education." A feature of the Colloquium Luncheon on Friday, June 16th, was the awarding of prizes for the Annual Exhibit of New Devices. PROFESSOR L. A. TURNER, *State University of Iowa*, presided at the luncheon and DR. EDWARD TELLER, *Los Alamos Scientific Laboratory*, led an informal discussion concerning the most important aspects of the relations of government and research.

The program follows:

Some problems in radiation biology. ROBERT L. SINSHEIMER, *Iowa State College.* TITUS C. EVANS, presiding.

Research reports: Investigation of the properties of thin metallic films—A. H. WEBER, *St. Louis University;* **Color perception and color deficiencies**—Z. V. HARVALIK, *University of Arkansas;* **Cold cathode mercury arc**—PAUL L. COPELAND, *Illinois Institute of Technology;* **Glow to arc transition**—J. G. WINANS, *University of Wisconsin.* H. K. SCHILLING, *The Pennsylvania State College*, presiding.

Paleotemperatures of the cretaceous. HAROLD C. UREY, *University of Chicago.* J. W. BUCHTA, *University of Minnesota*, presiding.

Some chemical evidence relative to the origin of the earth. HAROLD C. UREY, *University of Chicago.*

The value of physics history to nonscience major students. DUANE E. ROLLER, *Wabash College.* A. L. JOHNSON, *Crete, Nebraska*, presiding.

Laboratory round table. WILLARD J. POPPY, *Iowa State Teachers College;* PENROSE S. ALBRIGHT, *University of Wichita;* L. B. HAM, *University of Arkansas;* FRANK VERBRUGGE, *Carleton College;* LOUIS R. WEBER, *Colorado A. and M. College;* and R. L. EDWARDS, *Miami University.* O. H. SMITH, *DePauw University*, presiding.

Annual exhibit of new devices by members. Exhibitors included R. S. ALEXANDER, *Washburn Municipal University;* F. E. CHRISTENSEN, *University of Minnesota;* C. A. CULVER, *Park College;* J. A. ELDRIDGE, *State University of Iowa;* J. K. FLEMING, *United States Naval Academy;* P. E. FOSSUM, *St. Olaf College;* Z. V. HARVALIK, JOHN A. DOUGHTY, AND PAUL E. DAMON, *University of Arkansas;* W. J. HOOPER, *The Principia;* H. C. JENSEN, *Lake Forest College;* PAUL E. MARTIN, *Wheaton College;* MARVIN J. PRYOR, *New York State College for Teachers;* R. A. ROGERS, *Iowa State Teachers College;* PAUL ROOD, *Western Michigan College of Education;* J. L. RYERSON, *Evansville College;* JOHN S. SABY, *Cornell University;* A. F. SILKETT, *University of Illinois;* ANCIL R. THOMAS, *Valparaiso University;* FRANCIS E. THROW, *Cornell College;* AND P. W. WILLIAMS, *South Dakota State College.*

Interference colors reflected by very thin films. KATHERINE B. BLODGETT, *General Electric Research Laboratory.* P. S. HELMICK, *Drake University*, presiding.

The photographic emulsion in nuclear research. JOHN SPENCE, *Eastman Kodak Company Research Laboratories.* JAMES JACOBS, presiding.

The Associated June Lectures: Hydrodynamics in cosmic physics. EDWARD TELLER, *Los Alamos Scientific Laboratory.* 1. Turbulence and origin of the planetary system. 2. Shock hydrodynamics. 3. Magneto hydrodynamics. 4. Origin of cosmic rays. These lectures were sponsored by the Research Corporation.

NOTES AND DISCUSSION

A Quantitative Experiment on Rotational Motion*

PHILIP A. CONSTANTINIDES
Wilson Branch, Chicago City College, Chicago, Illinois

THE purposes of this experiment are: (a) to show that constant angular acceleration is imparted to a rotating body by a given torque, and that it varies proportionally with the applied torque; (b) to obtain the moment of inertia of the body from its rotational characteristics; and (c) to obtain the moment of inertia of the same body from its mass distribution, and to familiarize the student with approximations and methods of approach not encountered when bodies of simple geometrical shape are used.

1. Determination of Angular Acceleration.—To show that the angular acceleration imparted to a rotating body is proportional to the applied torque it suffices to prove experimentally that the corresponding peripheral accelerations are proportional to these torques. For these experiments a ball bearing bicycle wheel 31.70 cm in radius, provided with a lead rim[1] so that its moment of inertia would be large in proportion to its mass, was used. The wheel (Fig. 1(a)) was carefully balanced and mounted by means of V-shaped right angle clamps C on large stands placed on laboratory tables so that its axle was about two and a half meters above the floor. To obtain the peripheral acceleration one end of a two and a half meter strip of waxed paper weighing 2.60 g was glued to the rim of the wheel, while to its other end, after the strip was wrapped around the wheel, was attached the paper clamp A and the pan B on which was placed the weight producing the rotation.

The strip moved freely between the adjustable spark points P that were connected to a spark timer producing 120 sparks per second, the intensity of which was controlled by means of a rheostat connected in series with the primary of the spark timer. The data for the determination of the peripheral acceleration of the wheel were obtained from measurements similar to those used for verifying Newton's second law of motion. In Table I are given the experimentally determined values of accelerations, the corresponding successive groups of spark intervals on the trace from which these values were obtained, and the corresponding accelerating weights in which the average effect of the weight of the paper strip was considered. The subscripts to the various symbols refer to corresponding traces, while P_0 representing the initial point of each trace is selected so that succeeding points P_1, P_2, etc., are several millimeters from each other.

The constancy of the tabulated values of acceleration for a given trace proves that a constant torque imparts to a body constant angular acceleration, while the equality of accelerations obtained under identical torque conditions, but with different traces, gives a measure of the reliability of the experiment.

To determine the proportionality of imparted angular acceleration to the applied torque, let F_1 and F_3 represent the tangential forces exerted by the falling weights w_1 and w_3, on the periphery of the wheel when traces No. 1 and No. 3 were obtained and L_1 and L_3 the corresponding torques. Then, from the data in the table we have for the ratio of applied torques

$$\frac{L_3}{L_1}=\frac{F_3}{F_1}=\frac{w_3(g-a_3)}{w_1(g-a_1)}=\frac{1081(980.3-259.1)}{581.3(980.3-158.9)}=1.636, \quad (1)$$

while for the ratio of the corresponding angular accelerations, we have

$$\frac{\alpha_3}{\alpha_1}=\frac{a_3}{a_1}=\frac{259.1}{158.9}=1.631. \quad (2)$$

Similarly, from the data of traces No. 2 and No. 4, we have

$$\frac{L_4}{L_2}=\frac{F_4}{F_2}=\frac{1081(980.3-258.8)}{581.3(980.3-158.5)}=1.635 \quad (3)$$

and

$$\frac{\alpha_4}{\alpha_2}=\frac{a_4}{a_2}=\frac{258.8}{158.5}=1.633. \quad (4)$$

From the equality of the ratios of Eqs. (1) and (2) and that of Eqs. (3) and (4) we have convincing experimental evidence of the proportionality of angular accelerations to the applied torques.

2. Determination of Moment of Inertia.—The moment of inertia of the wheel was obtained from the relation

$$I=m(g-a)R^2/a. \quad (5)$$

Thus, using the value obtained from trace No. 1, we have

$$I_1=581.3(980.3-158.9)(31.70)^2/158.9 = 3{,}020{,}000 \text{ g cm}^2. \quad (6)$$

TABLE I. Data of peripheral acceleration of the wheel.

Accelerating weights $w_1=w_2=581.3$ g			Accelerating weights $w_3=w_4=1081$ g		
	Acceleration (cm/sec²)			Acceleration (cm/sec²)	
spark groups on trace	Trace No. 1	Trace No. 2	spark groups on trace	Trace No. 3	Trace No. 4
P_{120}, P_{60}, P_0	159.5	158.4	P_{80}, P_{40}, P_0	260.3	258.5
P_{130}, P_{70}, P_{10}	159.3	157.7	P_{90}, P_{50}, P_{10}	260.3	259.6
P_{140}, P_{80}, P_{20}	159.0	158.6	P_{100}, P_{60}, P_{20}	258.5	259.2
P_{150}, P_{90}, P_{30}	158.2	158.8	P_{110}, P_{70}, P_{30}	259.2	260.1
P_{160}, P_{100}, P_{40}	158.5	159.0	P_{120}, P_{80}, P_{40}	257.5	256.7
Av. accl. (cm/sec²)	$a_1=158.9$	$a_2=158.5$		$a_3=259.1$	$a_4=258.8$
Moment of inertia (g cm²)	$I_1=3{,}020{,}000$	$I_2=3{,}028{,}000$		$I_3=3{,}024{,}000$	$I_4=3{,}028{,}000$

Practically identical values were obtained from the other traces as can be seen from the tabulated results, the average value of which is $I=3{,}025{,}000$ g cm^2.

3. Calculated Moment of Inertia.—The moment of inertia of the wheel was evaluated by adding the moments of inertia of its various parts obtained from direct measurements of their position, dimensions and masses. The various elements of the wheel whose dimensions are given in Fig. 1(b) have the following masses:

Total mass of the rotating parts of the wheel $M=3328$ gm
Mass of 36 nipples, from the weight of one, $m_2=$ 49 g
Mass of 36 spokes, from the weight of one, $m_3=207$ g
Mass of hub from its dimensions (approx.), $m_4=$ 75 g
Mass of hoop $m_1=M-(m_2+m_3+m_4)=2997$ g.

The moment of inertia of the hoop, a cross section of which is given in Fig. 1(c), can be calculated by means of the parallel axis theorem after the center of gravity of a section of the hoop has been determined. The center of gravity of a section[2] is determined by balancing the section in two or more positions on the edge of a safety razor blade and determining the point of intersection of equilibrium lines obtained by pressing the section when in equilibrium on the edge of the blade. This point was found to be 0.54 cm beneath the periphery of the wheel. Thus, the centroidal distance d of the section from the axis of rotation CC is 31.16 cm.

Denoting by I_G the moment of inertia of the section with respect to the centroidal axis parallel to the CC-axis and by I_C the moment of inertia of the same section with respect to the CC-axis, we have from the parallel axis theorem

$$I_C=I_G+md^2. \tag{7}$$

But, since I_G from the mass distribution of the section is less than two-tenths of one percent of md^2 we may take

$$I_C=md^2=mK^2 \tag{8}$$

from which we have for the radius of gyration of the section, $K=31.16$ cm. Thus, the moment of inertia of the hoop $=mK^2=(2997)(31.16)^2=2910000$ gm cm^2. Since the radial distance of the nipples from the axis of rotation is $r=l\sin\theta=30$ cm, where l is the length of the spoke, we have

Moment of inertia of nipples
$$=m_2r^2=49(30)^2=44{,}100 \text{ g cm}^2.$$
Moment of inertia of spokes $=\frac{1}{3}m_3(l\sin\theta)^2$
$$=\tfrac{1}{3}m_3r^2=\tfrac{1}{3}207(30)^2=62{,}100 \text{ g cm}^2.$$
Moment of inertia of hub is negligible.

Therefore, moment of inertia of wheel $I=3{,}016{,}000$ g cm^2.

This value differs by 0.3 percent from the average value of moment of inertia obtained experimentally. The calculated moment of inertia, due to the limitations in the precision of evaluating the mass of the hub of the wheel and of the radius of gyration is not as precise as that obtained from the rotational characteristics of the wheel, but since it is based upon the very definition of moment of inertia, it gives a value which can serve as a fundamental reference by means of which the validity of the results obtained by other methods can be checked. The writer finds this method to be quicker and more precise than the one involving goniometric measuring devices and that the use of a more impressive object, such as the bicycle wheel, enhances the significance of the experiment. Much time can be saved if specimens of spokes, nipples and hub are available for direct determination of their masses and the location of the center of gravity of the section is given to the students.

In conclusion, I wish to express my thanks to John P. Karbler of the Physical Science Department for his help and interest in developing this experiment.

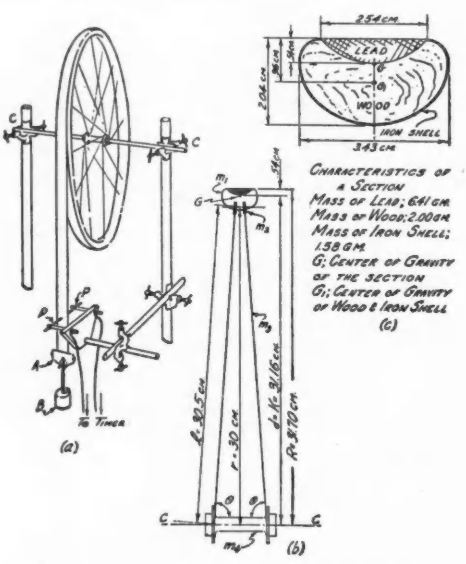

FIG. 1. Experimental arrangement of apparatus and dimensions of the wheel.

* This paper was read in the following meetings: Physics Section, Illinois Academy of Science, May 7, 1948; Chicago Chapter of A.A.P.T. December 4, 1948.
[1] This type of wheel is standard apparatus used for the demonstration of gyroscopic motion.
[2] This section may be replaced and soldered on the wheel or glued depending on the use to be made of the wheel.

Foreign Language for the Physics Student

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OFFERINGS of several foreign languages are now available to students in the grades, in the high schools and in the colleges. The author has investigated the need or desirability of language for the undergraduate student of physics by determining the requirements of the fifty-four graduate schools which enroll twenty-five or more

graduate students in physics.[1] Information has also been obtained regarding the advisability of language from thirty-three industrial corporations which employ physicists, since the American Chemical Society requires that a student show proficiency in German in its undergraduate program although there is no similar requirement in physics curricula or in the other sciences. The graduate group is considered first.

Foreign Languages in M.S. and Ph.D. Curricula.—Some graduate schools, such as Cornell University, require for the Master of Arts degree only a transcript showing two units in two languages or three units in one language. A unit represents one year's study, consisting approximately a quarter of a full-load year's work. For the Master's degree, the requirement of one language is most prevalent in the East, including New York state. Sixty percent of the twenty-one graduate schools consulted in this area have one language as a requirement. The schools having this requirement include the New England colleges and the older private institutions. In half of these schools no mention is made of a substitute language for either German or French, although Spanish, Italian and Russian may be substituted by petition in many colleges. The demand for a language for the M.S. degree decreases as one proceeds toward the middle west. Of the eighteen colleges investigated between New York state and the Mississippi River, only twenty-seven percent require one language. This percentage rises slightly to thirty-seven in the mountain and Southwestern regions where the number of institutions concerned is smaller and drops to thirty-three percent for the west coast colleges. In general, a candidate for the first graduate degree may need one language if he plans to enter a private university in the East. The student who anticipates the doctorate must prepare himself in two foreign languages. One-quarter of the total number of graduate schools make no mention of substitutes for German and French for this degree. Over half of the group lie in the east coastal states. In the remaining universities the other languages mentioned above are listed as permitted or may be substituted upon petition. Often this selection rests upon the major department. Over a third of these universities recognize Russian without petition; in fact, in only those institutions where German and French are definitely required is Russian not accepted.

Foreign Language Requirements for Scientists in Industry.—In recent discussions with industrial corporations which employ physicists the author included the question of foreign language. Half of the thirty-three corporations which replied either employ only physicists with the doctorate or did not express themselves. One-third of the remainder are in the negative group. These replies indicate that availability of translations make it possible for the student to spend the time more profitably in other fields. Some men in industry feel that the student often fails to become sufficiently grounded in the amount of language he studies in college and thus wastes his time because the language is not really useful to him. Concern is expressed over the so-called "scientific" language course which may make a man feel that he can actually read the language when he is likely to misread and misunderstand anything but the simplest writing because of his lack of basic knowledge of grammar. The other two-thirds prefer that the student study foreign language even though his industry may have adequate library facilities in which translations of articles may be made much more economically than having a technical man spend time in this manner. In many cases the feeling exists that language is advisable from the cultural point of view and should be studied by the prospective employee who hopes to become more than a "hands and foot" worker. One suggestion was the introduction of foreign language texts and reference materials in advanced courses to give practice in applying the language learned. Naturally where foreign service is a possibility, a fluent knowledge of language is desired, going beyond the ability merely to read and to translate. German is, by far, the language mentioned most frequently. French is considered less important and much easier for a student to acquire subsequently. There is a tendency, perhaps stated with a little hesitancy by several industries, toward the possibility that a knowledge of Russian may be necessary for the future.

[1] *Am. J. Physics* 17, 80 (1949).

Extensions of the Elementary Laboratory Experiment on Simple Harmonic Motion

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Dillard University, New Orleans, Louisiana

THE usual run of general physics laboratory experiments offers little intellectual excitement to the best students, and it is not uncommon to have them remark quite disparagingly on the questionable worthwhileness of some experiments. This is more often true in mechanics, so it seems, since so many of the experiments are fixed to a routine cookbook order of procedure. It appears instructionally sound, then, to extend an experiment, where possible, to more exciting and stimulating inquiry. The experiment on simple harmonic motion lends itself beautifully to such an extension.

The classical study is well known. Hooke's Law is investigated for a conical coil spring, the spring constant is found, and the experimental and calculated periods of a given suspended mass are compared. The student is told that to obtain the correct period we must add to the load on the spring one-third of the mass of the spring itself. In this whole business there is little to tax the imagination of the intellectually curious. The following extensions inject some interesting thinking and speculation. Indeed, I know of no other elementary experiment in mechanics that gives rise to such heated debate and excited discussion.

1. Find experimentally the spring constants k_1 and k_2 for two separate coil springs.
2. Speculate on the spring constant k for the system of the two springs connected *in series*, as in Fig. 1. The analytical student, with a little help, can

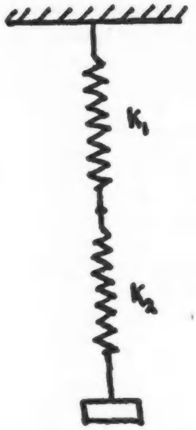

FIG. 1. Springs in series.

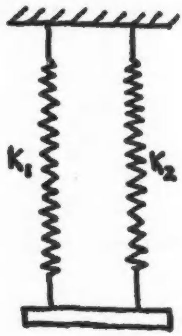

FIG. 2. Springs in parallel.

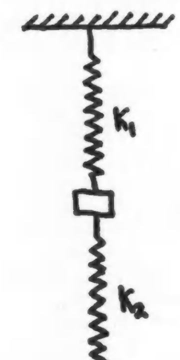

FIG. 3. Springs in parallel, a secondary arrangement.

establish

$$k=\frac{k_1k_2}{k_1+k_2}.$$

3. Check this k experimentally.
4. Repeat this for the two *parallel arrangements*, as in Fig. 2 and Fig. 3, and show $k=k_1+k_2$.
5. The motion of any of these systems (loaded with an arbitrary mass) may be timed and the periods compared with those given by the approximate formula. The differences here are quite substantial since the contribution of the spring masses to their own extension is not too easily established.

The alert student looks forward to analytical mechanics!

Isotope Shifts in the Balmer Spectrum of Tritium

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IN 1944 members of the Chemistry Division of the Oak Ridge National Laboratory prepared some tritium for research purposes, using the slow neutron capture by lithium for production. Previously tritium had been identified by its activity. To verify that the activity arose from hydrogen of mass three, the isotope shift of the Balmer lines was examined. From the theory of the hydrogenic atom, the fractional change in wavelength for corresponding lines from the different hydrogen isotopes is

$$\frac{\Delta\lambda}{\lambda}=-\frac{m}{M_1}\left(1-\frac{M_1}{M_2}\right),$$

where m is the electronic mass, and M_1, M_2, are nuclear masses. For comparison of lines from protium ($M_1=1$) and tritium ($M_2=3$), the fractional shift is $(1/1837)\times(1-\frac{1}{3})$ or 1/2755; for protium and deuterium ($M_2=2$) the fractional shift is three-quarters of this value. For the α-line of the Balmer spectrum the displacement to shorter wavelengths is then 1.79A for deuterium and 2.39A for tritium, from H_α (6562.8A).

The α-lines from our first spectrogram of tritium and the reference iron arc are reproduced in Fig. 1. The spectrograph had a wavelength range that permitted only the first three lines of the Balmer series to be recorded with one exposure, and Fig. 2 shows these lines from a later sample. Geissler tubes of about 12 cm^3 capacity with a 2-mm capillary were filled with the gas at 2 mm of mercury pressure, and exposed for ten minutes using a 25,000-volt transformer at several milliamperes. The 1.5-m grating spectrograph gave a linear dispersion of 0.14 mm/A, or a separation of 0.34 mm for the α-lines of tritium and protium; a 50-micron slit was used for the hydrogen exposures. No

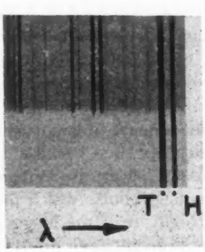

FIG. 1. First spectrogram of tritium (and protium; H_α at 6562.8A). Reference iron arc.

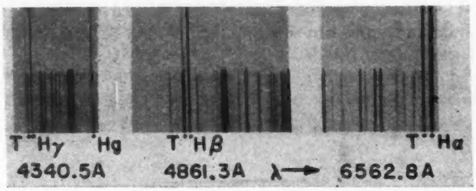

FIG. 2. Portions of a film showing the first three lines in the Balmer series of tritium and protium. Reference iron arc.

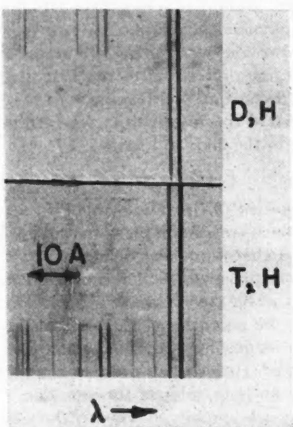

FIG. 3. Two spectrograms of the H_α lines juxtaposed to compare the tritium-protium wavelength difference with the deuterium-protium difference. Reference iron arc.

Geissler tubes were ever filled with all three isotopes of hydrogen, but for visual comparison a composite picture, Fig. 3, was made by matching two spectrograms made at different times. From measurements of a film exposed with a slit 10 microns wide, which corresponded to a line breadth of 0.07A, the tritium shift was found to be 2.36±0.05A (estimated error) from H_α (6562.8A).

Letters to the Editor

On the Derivation of Euler's Equation for the Motion of an Inviscid Fluid

IN a recent article on hydrodynamics[1] it was found necessary to neglect the time derivative of the fluid density in order to arrive at the correct form of Euler's equation [Eq. (6) in reference 1] for the motion of a compressible fluid! This came about because the "dynamic equation" [Eq. (5) in reference 1] was derived incorrectly.

The two most common methods of deriving Euler's equation may be outlined as follows:

1. Use a *given fluid mass* as "free body," and express conservation of momentum in the classical elementary form

time rate of change of momentum = net force.

2. Use a fixed closed *control surface*, stationary with respect to the coordinate system. Then fluid flows continuously through the control surface, and we apply conservation of momentum in the form

time rate of increase of momentum contained within the fixed control surface = net force + net rate at which momentum is carried into the control surface by convective velocities.

Method 1 is employed in many texts,[2] and leads directly to the form

$$\rho\frac{\partial\mathbf{V}}{\partial t}+\rho(\mathbf{V}\cdot\nabla)\mathbf{V}=\rho\mathbf{g}-\nabla p; \tag{1a}$$

or, in Cartesian tensor notation,

$$\rho\frac{\partial u_i}{\partial t}+\rho u_k\frac{\partial u_i}{\partial x_k}=\rho g_i-\frac{\partial p}{\partial x_i}. \tag{1b}$$

Method 2 is less common.[3] In vector notation it requires dyadics; hence the Cartesian tensor notation is perhaps more suitable. This method leads directly to the form

$$\frac{\partial}{\partial t}(\rho u_i)+\frac{\partial}{\partial x_k}(\rho u_i u_k)=\rho g_i-\frac{\partial p}{\partial x_i}, \tag{2}$$

where a repeated index signifies summation.

Equations (1) and (2) are equivalent forms, connected simply by the equation for conservation of mass:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_k}(\rho u_k)=0. \tag{3}$$

In reference 1, the author apparently set out to use Method 1, but failed to include the time rate of change of the volume containing the fixed mass of fluid.

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[1] J. B. Kelley, *Am. J. Physics* 18, 202 (1950).
[2] See for example, J. G. Coffin, *Vector analysis*, 2nd ed. (J. Wiley & Sons, Inc., New York, 1911), p. 210.
[3] See for example, H. W. Liepmann and A. E. Puckett, *Introduction to aerodynamics of a compressible fluid*, 1st ed. (J. Wiley & Sons, Inc., New York, 1947), p. 108 *et seq.*

The Extended Bernoulli Equation

SEVERAL unfortunate errors crept into my paper[1] and were the result of hasty checking on my part between the rough draft and the typed copy sent to the *Journal.* In the final copy the parentheses on the left-hand side of Eq. (4) were misplaced. In the rough draft, Eq. (4) through the statement after Eq. (6) read

$$\frac{d}{dt}(\rho v dxdydz)=g(\rho dxdydz)-\nabla P(dxdydz). \tag{4}$$

Dropping the common term from Eq. (4) gives

$$\frac{dv}{dt}\rho=g\rho-\nabla P. \tag{5}$$

This is the *dynamic equation*. It is also called *Euler's equation* when written in the form

$$\frac{dv}{dt}=g-\frac{\nabla P}{\rho}. \qquad (6)$$

Equation (6) implies the assumption that

$$v\frac{d}{dt}(\rho dxdydz)$$

is negligible (zero).

In criticism of above, which is correct, I should like to say that Eq. (6) is superfluous and the statement after Eq. (6) is ambiguous. It would have been better to say "Equation (5) includes the fact that

$$v\frac{d}{dt}(\rho dxdydz)$$

is zero (since the mass is assumed to be constant by the law of conservation of mass) and therefore $\rho dxdydz$ is a multiplier, not a function of t, and hence may be divided out."

However, the equations which follow are all correct. But Eq. (14d), apparently through an understandable printing error, reads

$$\frac{1}{\rho}(\rho-\rho_0)\text{ etc.,}$$

whereas it should read

$$\frac{1}{\rho}(p-p_0)\text{ etc.}$$

I should like to thank those, including one of my colleagues here at Hofstra, who pointed out the inconsistencies in Eqs. (4) and (5) and the bad descriptions which followed these equations. I should also like to absolve James A. Moore of any blame in this matter because the rough draft he checked was correct.

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[1] James B. Kelley, "The extended Bernoulli equation," *Am. J. Physics* **18**, 202 (1950).

Relative Densities of Sun and Moon

A SIMPLE dimensional argument can be used to show that the density of the sun is less than that of the moon. The tidal effect of a remote object is proportional to the difference[1] in its gravitational force on the two sides of the earth, or to dF/dR. Since gravitational force F is proportional to M/R^2, the tidal effect is proportional to M/R^3. We now use two well-known observations: (1) The moon tides are stronger than the sun tides, and (2) The angular diameter of the sun and moon are almost identical, as shown by an eclipse of the sun. If the sun and moon had the same densities, their masses would be proportional to the cubes of their distances, from (2), and their tidal effects would be equal. Since the tidal effect of the moon is greater, the density of the moon must be greater than that of the sun.

(The Encyclopedia Britannica gives 2.17 as the ratio of the moon and sun tide amplitudes, and $3.33/1.41=2.35$ as the ratio of the densities.)

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[1] E. Mach, *The science of mechanics* (Open Court, Ed. 5, 1942), pp. 259–264.

Maxwell's Equations, Not Again!

YES, here is another method for recalling Maxwell's thermodynamical relations. Write the abbreviation of the plural of the word *private* to emphasize P, V, T, and S. Place the differential of the first quantity, i.e., P (the 2nd held constant) with respect to the 4th equal to the differential of the 3rd quantity (the 4th held constant) with respect to the 2nd, and write it as

$$P_+(P_V)_{;S}=P_-(T_S)_{;V}, \qquad (1)$$

where $(P_V)_{;S}=(\partial P/\partial S)_V$. The symbol $P_\pm$ denotes a permutation of the quantities in the parenthesis; the subscript, minus, means that while the quantities on the right are being permuted those on the left are held constant in the parenthesis, and the converse is true for P_+. Permuting the quantities in the parenthesis does not change the symbol following the semicolon on the side where the operation is being performed. However, notice that the subscripts have inverse order on both sides of the equation and it is this order that must be preserved. Hence, the subscript inside the parenthesis will be changed where the operation is performed while on the other side of the equation only the symbol following the semicolon will change. Finally, a plus sign is associated with the equation containing P and S in respective positions on both sides; or if you like, associate the numbers 1, 2, 3, and 4 with P, V, T, and S, and if the sum of the quantities following the semicolon is odd, then a plus sign is inserted; if the sum is even, then a minus sign. Thus Eq. (1) becomes, omitting the parentheses,

$$-P_{V;S}=T_{S;V}. \qquad (2)$$

Permuting the quantities on the left of Eq. (2),

$$V_{P;S}=T_{S;P}. \qquad (3)$$

Permuting the quantities on the right of Eq. (3),

$$-V_{P;T}=S_{T;P}. \qquad (4)$$

Permuting the quantities on the left of Eq. (4),

$$P_{V;T}=S_{T;V}.$$

LeRoy Pietsch
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The Simple Pendulum

THE simple or ideal pendulum is defined as a point body suspended by a cord of negligible mass. The equation for its period of vibration may be derived (a) by equating the accelerating force on the bob to the product

of the mass and linear acceleration of the bob and then applying the equation for the period of a simple harmonic motion, or (b) by regarding it as a special case of the physical or compound pendulum. These derivations will be reviewed here with comments as to their relative values and applicability to the usual pendulum used in the general physics laboratory.

Let a material particle of mass m, suspended at the end of a massless cord of length L, be displaced a distance x from its equilibrium position. The accelerating force on the bob is mgx/L and the product of the mass and linear acceleration a of the bob is ma. Equating these two expressions and noting that the acceleration is opposite in direction to the displacement, we get $-x/a=L/g$. If the displacement is small compared with L then the motion of the particle is approximately simple harmonic and the general equation for the period of a simple harmonic motion, $T=2\pi(-x/a)^{\frac{1}{2}}$, is applicable. This gives for the period of the simple pendulum, $T=2\pi(L/g)^{\frac{1}{2}}$.

Even though the definition requires that the bob of the pendulum be a point mass there is nothing in this conventional derivation which prevents the use of a bob of any size whatever. The quantities involved in the derivation would remain the same for a given mass for any choice of size or shape of the bob and, of course, the formula for the period of the pendulum would not vary with the size of the bob. In this respect this derivation is inadequate since it is known that for a given length of the pendulum the period does vary with the size of the bob.

In the general physics laboratory, a metal sphere is frequently suspended by a string as an approximation to the simple pendulum. Since the moment of inertia of the sphere about the point of suspension is $(2mr^2/5)+mL^2$, where r is the radius of the sphere, we get for the period of the laboratory pendulum

$$T=2\pi[L/g((2r^2/5L^2)+1)]^{\frac{1}{2}}. \tag{1}$$

The period of the laboratory pendulum, therefore, depends upon the radius of the sphere as well as the length of the suspension cord. If in Eq. (1), $r=0$, i.e., if the bob is a point mass, then this equation reduces to $T=2\pi(L/g)^{\frac{1}{2}}$, the equation for the period of a simple pendulum.

This derivation makes clear the necessity for assuming that the simple pendulum bob is a point mass. Furthermore, the calculated value of the period of the laboratory pendulum by means of Eq. (1) checks more closely with experimental values than the value calculated from $T=2\pi(L/g)^{\frac{1}{2}}$.

An examination of twenty college textbooks of general physics reveals that all except one[1] use a derivation similar to (a) for the equation for the period of the simple pendulum. Of these textbooks approximately two thirds present a discussion of the simple pendulum after treating moment of inertia and torque. All of the textbooks examined treat the concept of moment of inertia in sufficient detail to provide the necessary background for an understanding of the derivation of the formula for the period of the physical pendulum. In view of these facts and the inadequacy of derivation (a) in the respect mentioned it seems desirable to present the simple pendulum as a special case of the physical pendulum.

G. Preston Burns

Mary Washington College of the University of Virginia, Fredericksburg, Virginia

[1] R. A. Millikan, D. Roller and E. C. Watson, *Mechanics, molecular physics, heat and sound* (Ginn and Company, New York, 1937), p. 343. Textbooks other than those of elementary college physics treating the simple pendulum as a special case of the physical pendulum include *Properties of matter* (Prentice-Hall New York, 1937) by Champion and Davy and *The general properties of matter* (Ernest Benn, London, 1928) by Newman and Searle.

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ANNOUNCEMENTS AND NEWS

Book Reviews

Out of My Later Years. Albert Einstein. Pp. 282+viii; Figs. 6+1 Plate; $5\frac{1}{4}\times 8\frac{1}{2}$ in. The Philisophical Library, Inc., New York, 1950. Price $4.75.

This is not, for the most part, a book about Physics. On the contrary, it is a collection of some 60 notes, essays, occasional addresses, and hitherto unpublished articles of very general interest which in their variety defy and overrun even such loose classifications as the publishers have attempted. The headings for these groupings are: *Convictions and Beliefs* (40 pp.); *Science* (80 pp.); *Public Affairs* (50 pp.); *Science and Life* (35 pp.); *Personalities* (20 pp.); *My People* (30 pp.). As indicated, these groupings divide but do not classify. Thus much of the material included under *Public Affairs* seems to fit equally well to the designation, *Convictions and Beliefs*. This is, however, quite immaterial. Except for the section on *Science*, where a definite continuity exists, the book should be treated as "pick-up" reading. In this way one avoids the irritation due to the violent mental dislocations involved in the continuous reading of discontinuous material. What is, however, of much greater importance; this method of fragmentation allows one to see each article as a revelation of some element in the personality or attitudes of the author. Thus one may achieve what the reviewer has found best in the book, a feeling of intimacy with a personality of wide

interests and deep sympathies. This one rarely gets from the printed page.

The section on *Science* may be highly recommended as added material for such students of first-year college physics as can be persuaded to venture outside of the textbook. High School teachers will also find it stimulating and may even wish to recommend it to some of their more ambitious pupils.

The thumb-nail portraits under *Personalities* form a most unique and interesting section of the book.

A. A. KNOWLTON
Bennington College

A Brief Course in Physics for Students of Home Economics. LESTER T. EARLS. Pp. 340. Prentice-Hall, Inc., New York, 1949. Price $5.65.

This textbook by Lester T. Earls is well written, the sketches are clear and descriptive, and the language well chosen considering the type of students to which the book addresses itself. Being used in a one-semester physics course, the book has to be brief. The author therefore has cut down the topics to a minimum; thus the teacher has no choice in his presentation of topics.

Subjects are discussed simply, and the book is very readable. Some parts, however, might need a more detailed discussion in order to give the student an understanding of the ideas involved. This, for example, is the case in the discussion of force (p. 22). The concepts of motion, uniform acceleration due to gravity, and inertia seem to be so important that they cannot be left out, even in the briefest and most elementary physics book, which, after all, addresses itself to college students. The same is true of Archimedes' principle and sound, all of which are not only important from the viewpoint of physics but also from the historical and cultural viewpoint. Conservation of energy, after being explained very clearly on p. 41, finds no practical application anymore, as, for instance, in the chapter on machines, where it could be illustrated to great advantage.

The discussion on heat on pp. 61, 62, and 64 uses concepts of electricity which, however, have not yet been discussed, and students might therefore have difficultity in understanding the implications. It might be easier to come back to the discussion of heat, after developing the necessary steps in electricity, thus giving also an opportunity to review heat and to show the correlation between the different branches of physics.

Electrical units (pp. 170, 176) are discussed so briefly that the student does not get acquainted at all with their meaning. Simple mathematical proofs, like the ones for resistors in series and parallel, are omitted, as well as the explanation of simple laboratory experiments.

The section on light is exceptionally clear, well illustrated, and most rewarding for teacher and student.

ANNA M. AKELEY
Purdue University

Live with Lightning. MITCHELL WILSON. Pp. 404. Little, Brown and Company, Boston, 1949. Price $3.00.

Most serious students of physics doubtless feel that the cultivation of their science is a glamorous occupation. It probably occurs to most such men that science has not yet been properly represented in fiction—that the public characterization of science and the scientist is ludicrously and harmfully wrong, and ought to be improved by someone who knows better.

Either fortunately or unfortunately, most physicists are not writers. Despite their undeniable dissatisfaction with the prose offered them and others by commercial writers, they themselves cannot repair it.

Mitchell Wilson is an exception. The jacket blurb of *Live with Lightning* assures us that he has engaged in wholly respectable graduate work in physics. At the same time, he has managed to sell, for money, works of fiction that he himself has written.

In *Live with Lightning*, he has produced the most authentic novel about a physicist that has yet been written. The limited enthusiasms and the sounds and dreams of the laboratory are faithfully recorded, with understandable and minor distortion. Still, there have been few novels about physicists, and this is faint praise.

Live with Lightning is the story of Erik Gorin and his growth into maturity. After graduate work at Columbia, Gorin tries everything. He is an instructor at Michigan, a staff member in a squalid Southern institution, an engineer for a machine-tool firm, and finally a leader in the wartime atomic-energy program. Along the way, he betrays his wife by the seduction of a handy female physicist.

The climax of the book comes near the end. The forces of public evil, as represented by the sharp lawyer who had earlier courted Gorin's wife, propose to capture atomic energy for exploitation by large and heartless corporations. Venal members of the Congress are supporting this attempt. A refugee physicist who has been close to Gorin sacrifices himself as a gesture of repugnance for such sordid enterprise. Gorin finds a letter which makes it clear that the dead man has been admiring Gorin's wife from afar.

Over the bier, Gorin finds himself again, and courts his wife once more. As Erik and Savina walk off, hand in hand, into the sunset, it is clear that life will offer future difficulties but, in the main, be happy.

This isn't nearly so bad a book as this short and unfair summary may indicate. Many things about it are good. But it is a disappointment that the first serious novel to explore the development of a physicist could not have been much better. The plot is far too obviously contrived to be convincing. The good men and the bad ones are far too surely painted in for us in white and black. Nevertheless, it is a pleasure to have a book about a physicist written by a physicist, even a retired one. Maybe there will be more of this in the future.

Meanwhile, it is altogether clear that atomic energy is too big a matter even for Mitchell Wilson and Erik Gorin put together.

LOUIS N. RIDENOUR
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New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Physics* **18**, 340 (1950)].

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